CS 261
Fall 2016

Mike Lam, Professor

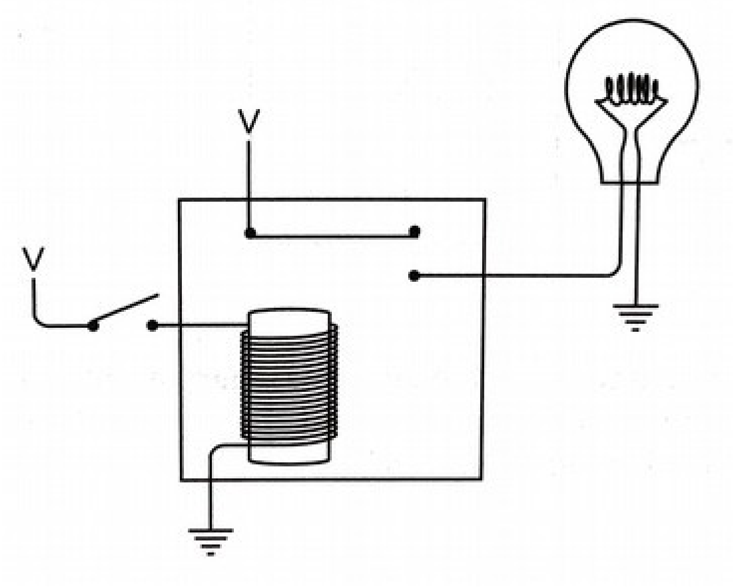
Logic Gates

The final frontier

- Java programs running on Java VM
- C programs compiled on Linux
- Assembly / machine code on CPU + memory
- ???
- Switches and electric signals

Aside: Relays

- From “Code” recommended reading:



Relay

Question: what happens if we connect the light bulb to the other contact?

Aside: Relays

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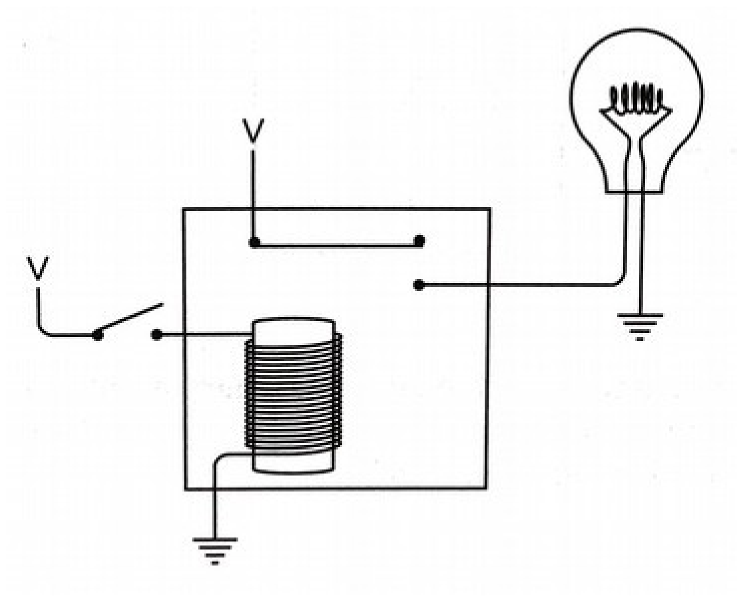
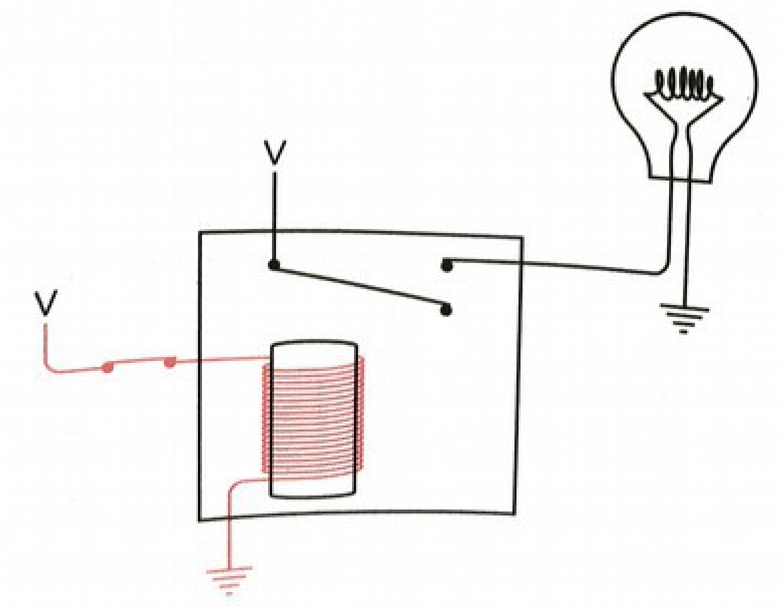


Regular relay



Inverted relay (NOT)

Aside: Relays

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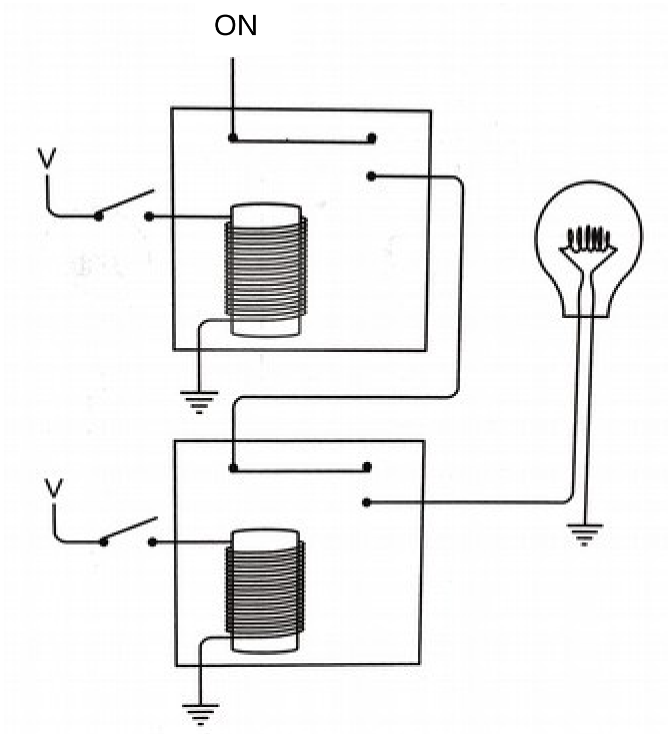

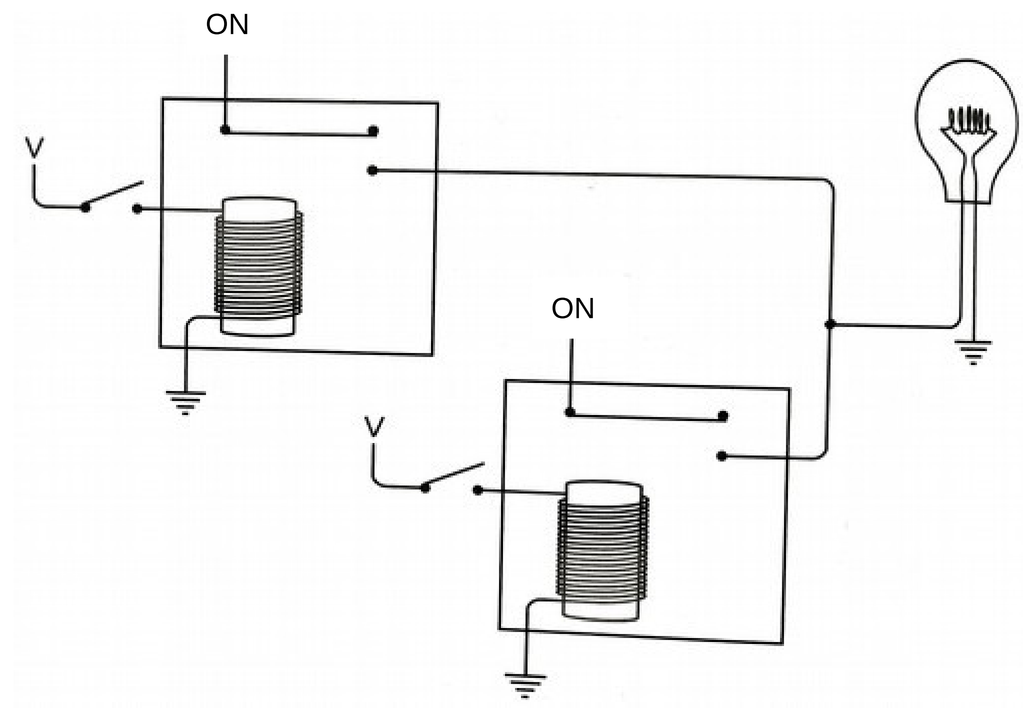

Aside: Relays

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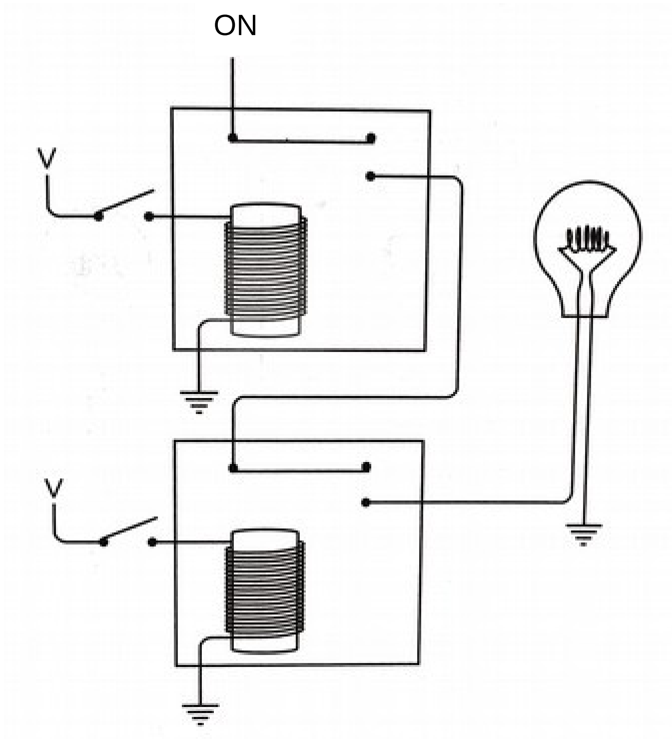
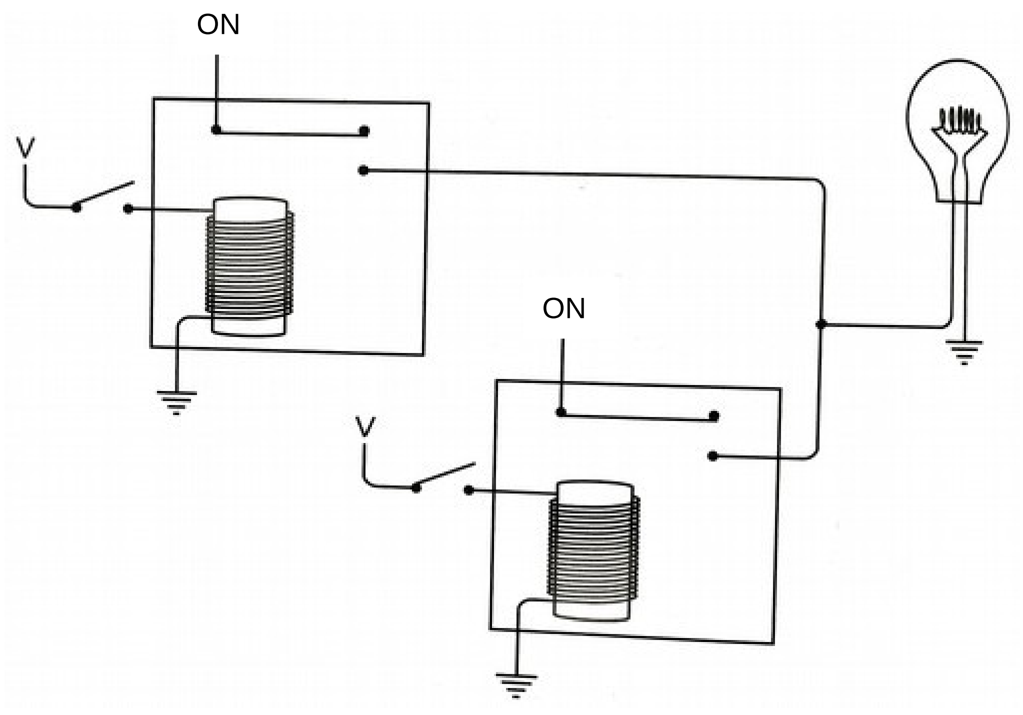


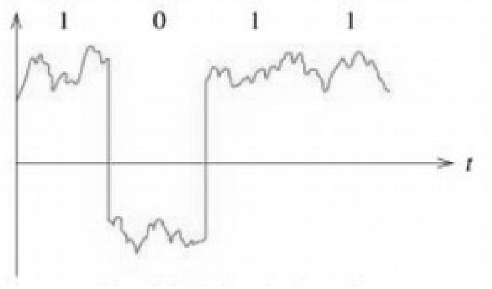
Relays in series (AND)

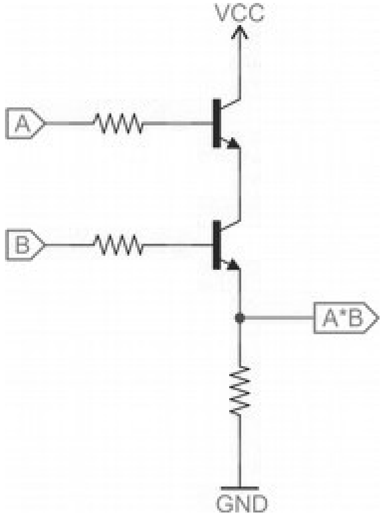


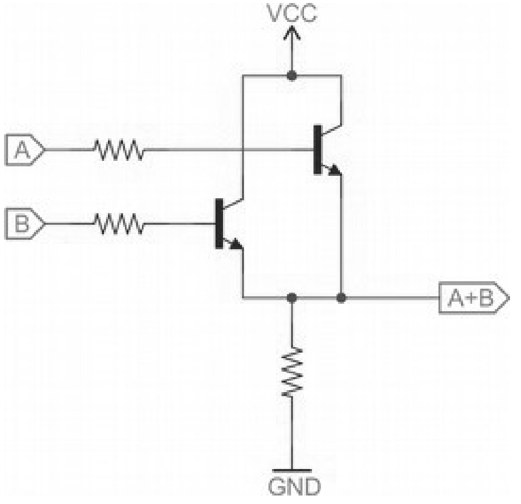
Relays in parallel (OR)

Digital hardware

- Digital signals are transmitted via electric signals by varying voltages
  - 1.0 V (high) = binary 1
  - 0.0 V (low) = binary 0
  - Use a threshold to distinguish







Images from https://learn.sparkfun.com/tutorials/transistors

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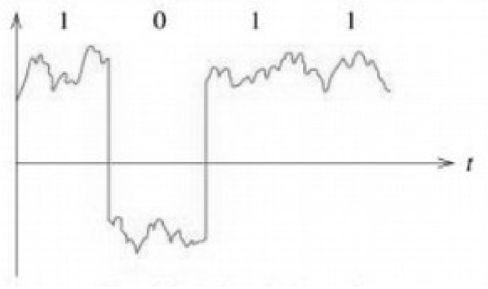
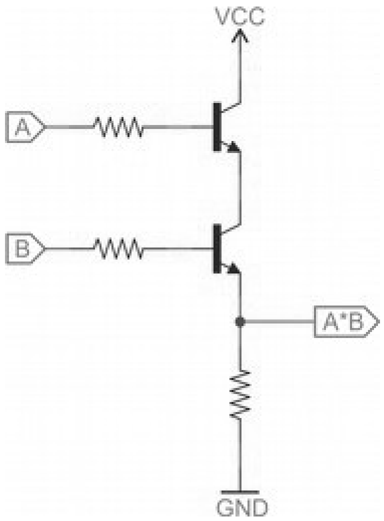
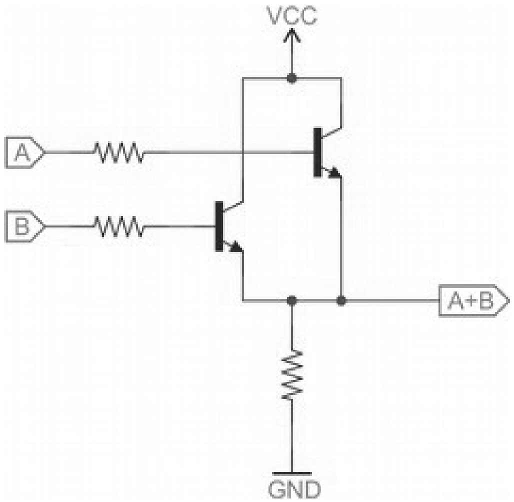




AND

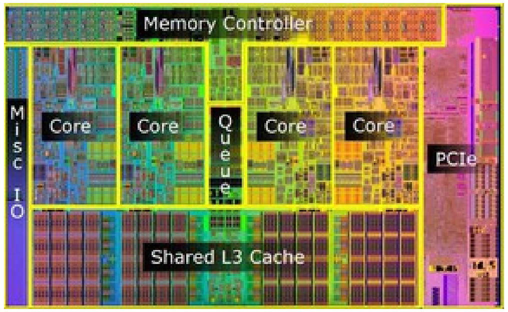


OR

Images from https://learn.sparkfun.com/tutorials/transistors

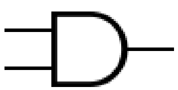
Transistors

- Transistors are the fundamental hardware component of computing
  - Similar to relays; replaced vacuum tubes
    - Smaller, more reliable, and use less energy
    - Primary functions: switching and amplification
  - Mostly silicon-based semiconductors now
    - Metal–Oxide–Semiconductor Field-Effect Transistor (MOSFET)
    - n-channel (“on” when V_{gate} = 1V) vs. p-channel (“off” when V_{gate} = 1V)
    - Mass-produced on integrated circuit chips
  - For convenience, we abstract their behavior using logic gates



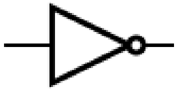
Logic gates

- Primary gates:

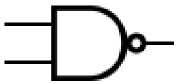


&	0	1
0	0	0
1	0	1

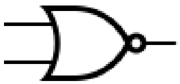
|	0	1
0	0	1
1	1	1



!	
0	1
1	0



	0	1
0	1	1
1	1	0



	0	1
0	1	0
1	0	0

^	0	1
0	0	1
1	1	0

Logic gates

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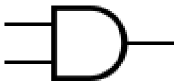
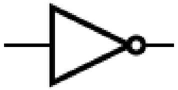


&	0	1
0	0	0
1	0	1

AND

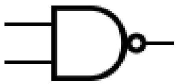
|	0	1
0	0	1
1	1	1

OR



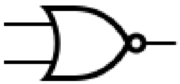
!	
0	1
1	0

NOT



	0	1
0	1	1
1	1	0

NAND



	0	1
0	1	0
1	0	0

NOR

^	0	1
0	0	1
1	1	0

XOR

Basic combinatorial circuits

- Circuits are formed by connecting gates together
  - Textbook uses Hardware Description Language (HDL)
  - Equivalent to boolean formulas or functions
    - f(g(x, y)) means apply “operation f to the result of operation g on x and y”
    - In a diagram: x,y → g → f (i.e., ordering is g first, then f)

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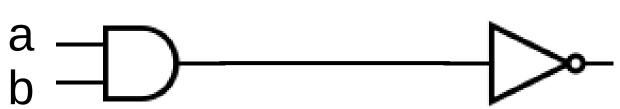
  - NAND example: (similarly for NOR)
    - Infix/boolean notation: **a NAND b = !(a & b)**
    - Function notation: **NAND(a, b) = NOT(AND(a, b))**



	0	1
0	0	0
1	0	1

a AND b

	0	1
0	?	?
1	?	?

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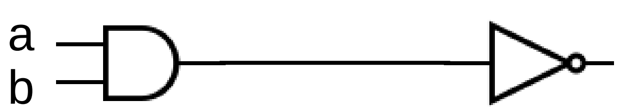
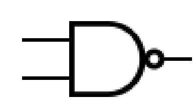


	0	1
0	0	0
1	0	1

a AND b

	0	1
0	1	1
1	1	0

NOT (a AND b)

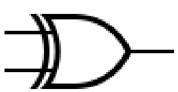


	0	1
0	1	1
1	1	0

a NAND b

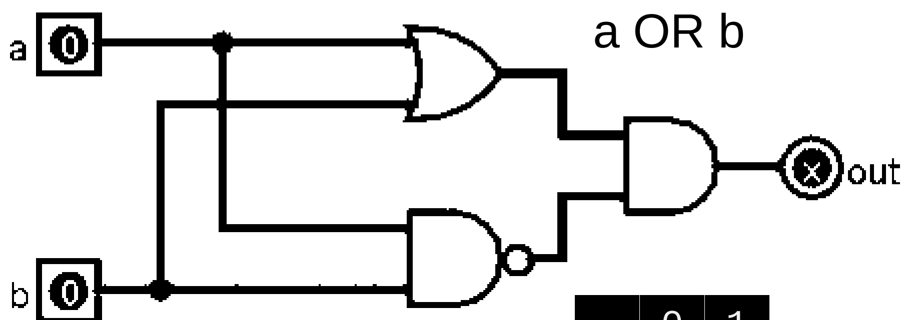
Basic combinatorial circuits

- Circuits are equivalent if the truth tables are the same
  - **a XOR b = (a OR b) AND (a NAND b)**
  - **XOR(a, b) = AND(OR(a,b), NAND(a,b))**



^	0	1
0	0	1
1	1	0

XOR



|	0	1
0	0	1
1	1	1

a OR b

	0	1
0	1	1
1	1	0

a NAND b

	0	1
0	?	?
1	?	?

(a OR b) AND (a NAND b)

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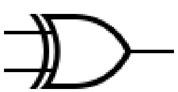
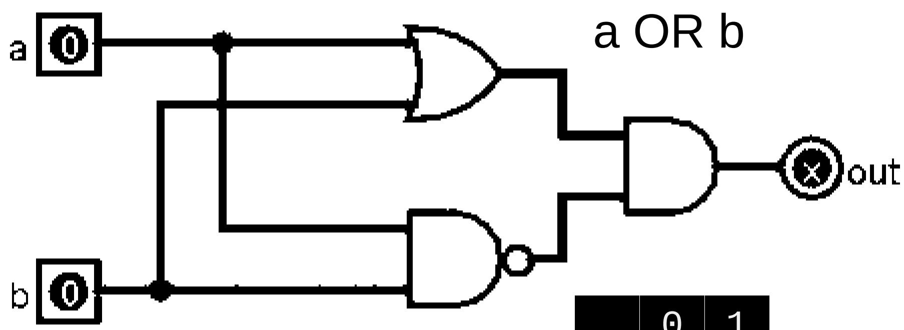


^	0	1
0	0	1
1	1	0

XOR



|	0	1
0	0	1
1	1	1

a OR b

	0	1
0	1	1
1	1	0

a NAND b

	0	1
0	0	1
1	1	0

(a OR b) AND
(a NAND b)

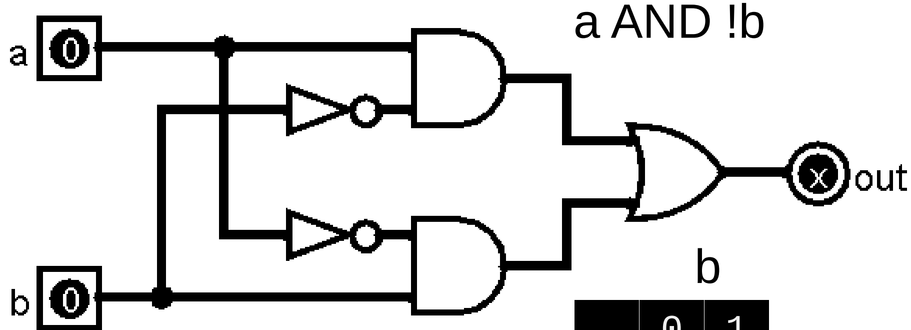
Basic combinatorial circuits

- Circuits can be equivalent even if the structure is different
  - **f(a, b) = (a AND !b) OR (!a AND b)**
  - What is this equivalent to?

b

a	0	1
0	0	0
1	1	0

a AND !b



b

a	0	1
0	0	1
1	0	0

!a AND b

	0	1
0	?	?
1	?	?

(a AND !b) OR (!a AND b)

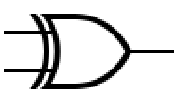
Basic combinatorial circuits

- Circuits can be equivalent even if the structure is different
  - **f(a, b) = (a AND !b) OR (!a AND b)**
  - What is this equivalent to?

b

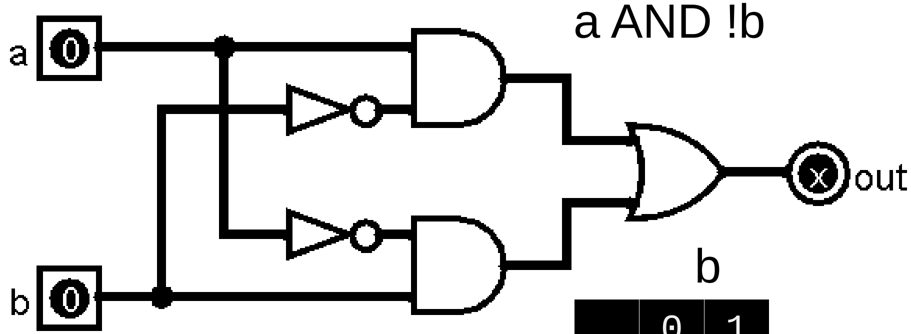
a	0	1
0	0	0
1	1	0

a AND !b



^	0	1
0	0	1
1	1	0

XOR



b

a	0	1
0	0	1
1	0	0

!a AND b

	0	1
0	0	1
1	1	0

(a AND !b) OR (!a AND b)

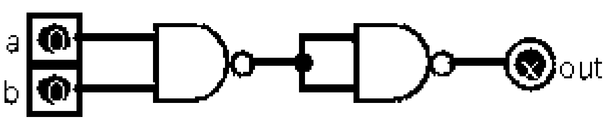
Important properties

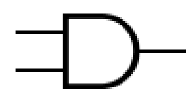
- Identity: **a AND 1 = a** **(a OR 0) = a**
- Constants: **a AND 0 = 0** **(a OR 1) = 1**
  - Also: **a NAND 0 = 1** **(a NOR 1) = 0**
- Inverses: **a NAND 1 = !a** **(a NOR 0) = !a**
  - Also: **a NAND a = !a** **a NOR a = !a**
- Double inverse: **!!a = a**
  - Or: **NOT(NOT(a)) = a**
- De Morgan's law: **!(a & b) = !a | !b**
  - Alternatively: **!(a | b) = !a & !b**

(remember this from CS 227?)

Universal gates

- NAND and NOR gates are universal
  - Each one alone can reproduce all other gates
  - Example: **a AND b** = a & b = !(!(a & b)) = !(a NAND b) = **(a NAND b) NAND (a NAND b)**





	0	1
0	1	1
1	1	0

a NAND b

	0	1
0	0	0
1	0	1

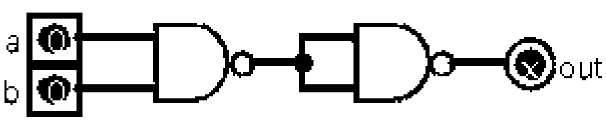
(a NAND b) NAND (a NAND b)

	0	1
0	0	0
1	0	1

a AND b

Universal gates

- NAND and NOR gates are universal
  - Each one alone can reproduce all other gates
  - Example: **a AND b** = a & b = !(!(a & b)) = !(a NAND b) = **(a NAND b) NAND (a NAND b)**
    - Similarly: **a AND b** = !(!(a & b)) = !(!a | !b) = !a NOR !b = **(a NOR a) NOR (b NOR b)**

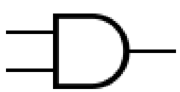


	0	1
0	1	1
1	1	0

a NAND b

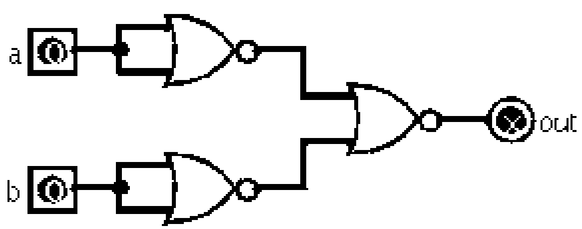
	0	1
0	0	0
1	0	1

(a NAND b) NAND (a NAND b)



	0	1
0	0	0
1	0	1

a AND b

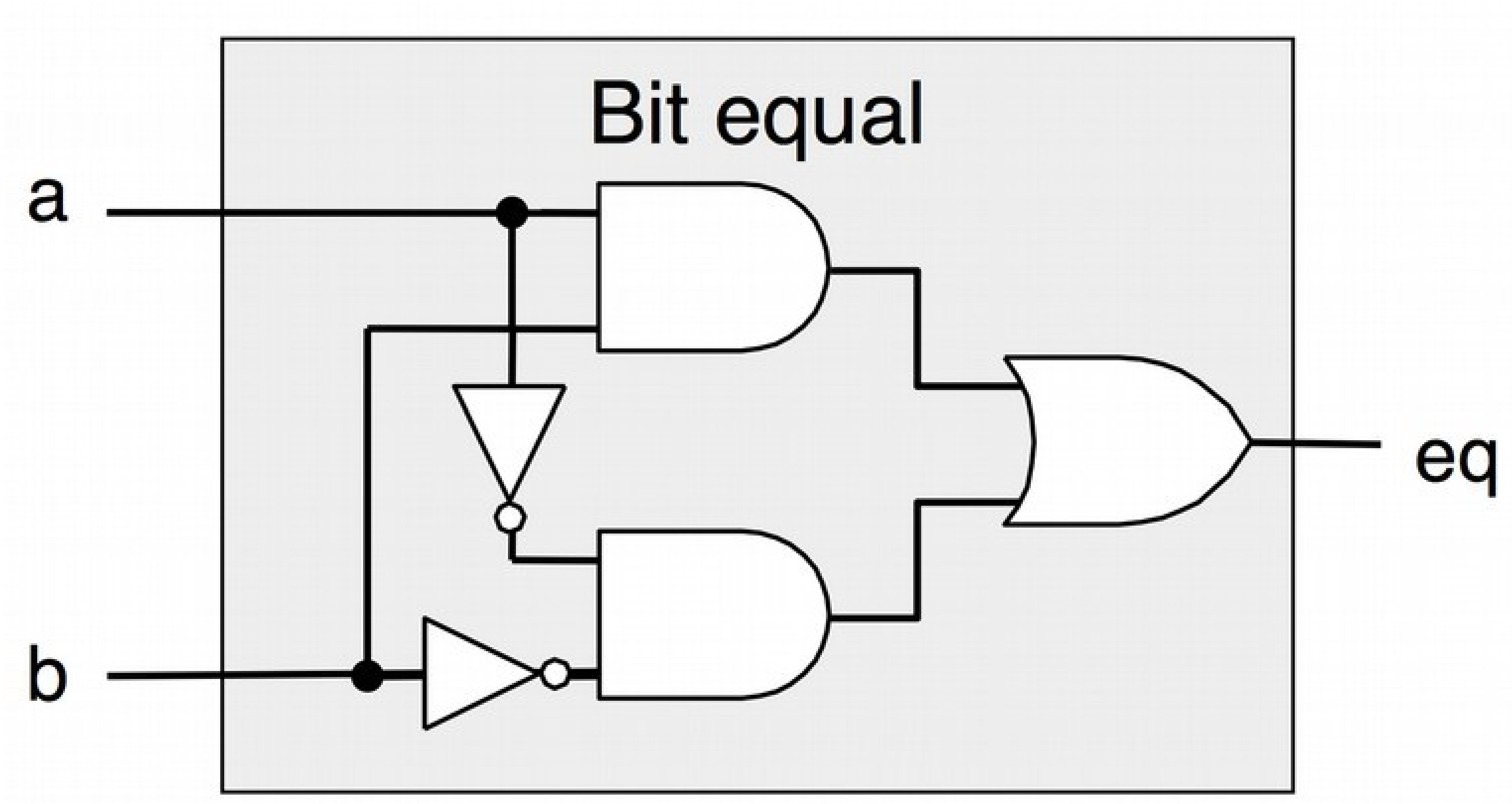


(a NOR a) NOR (b NOR b)

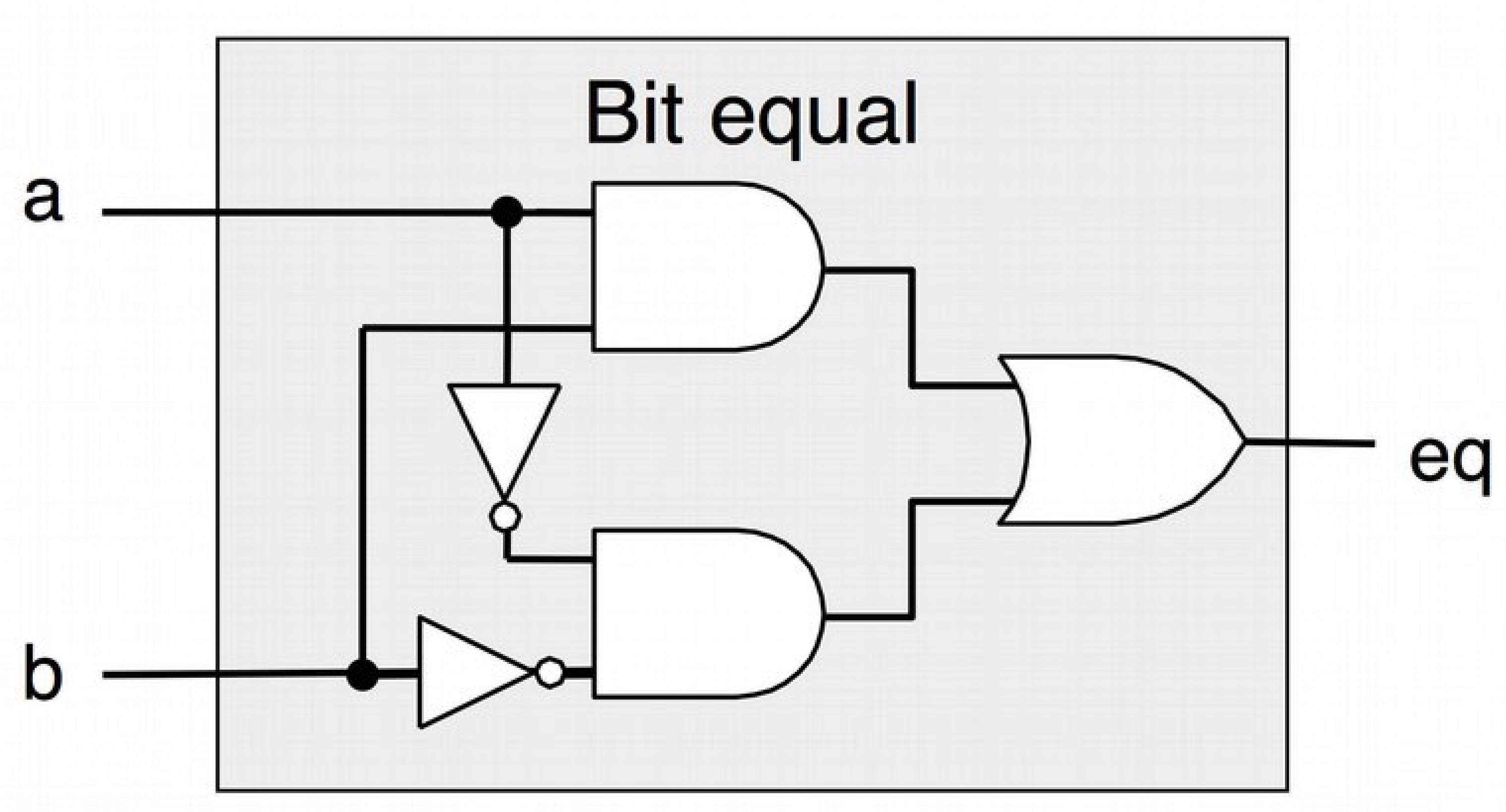
Computation

- Identify circuits that perform useful computation
  - Testing bits to see if they’re equal
  - Selecting between multiple inputs
  - Adding or subtracting bits
  - Bitwise operations (AND, OR, XOR)
  - Make them work on bytes instead of bits

Equality



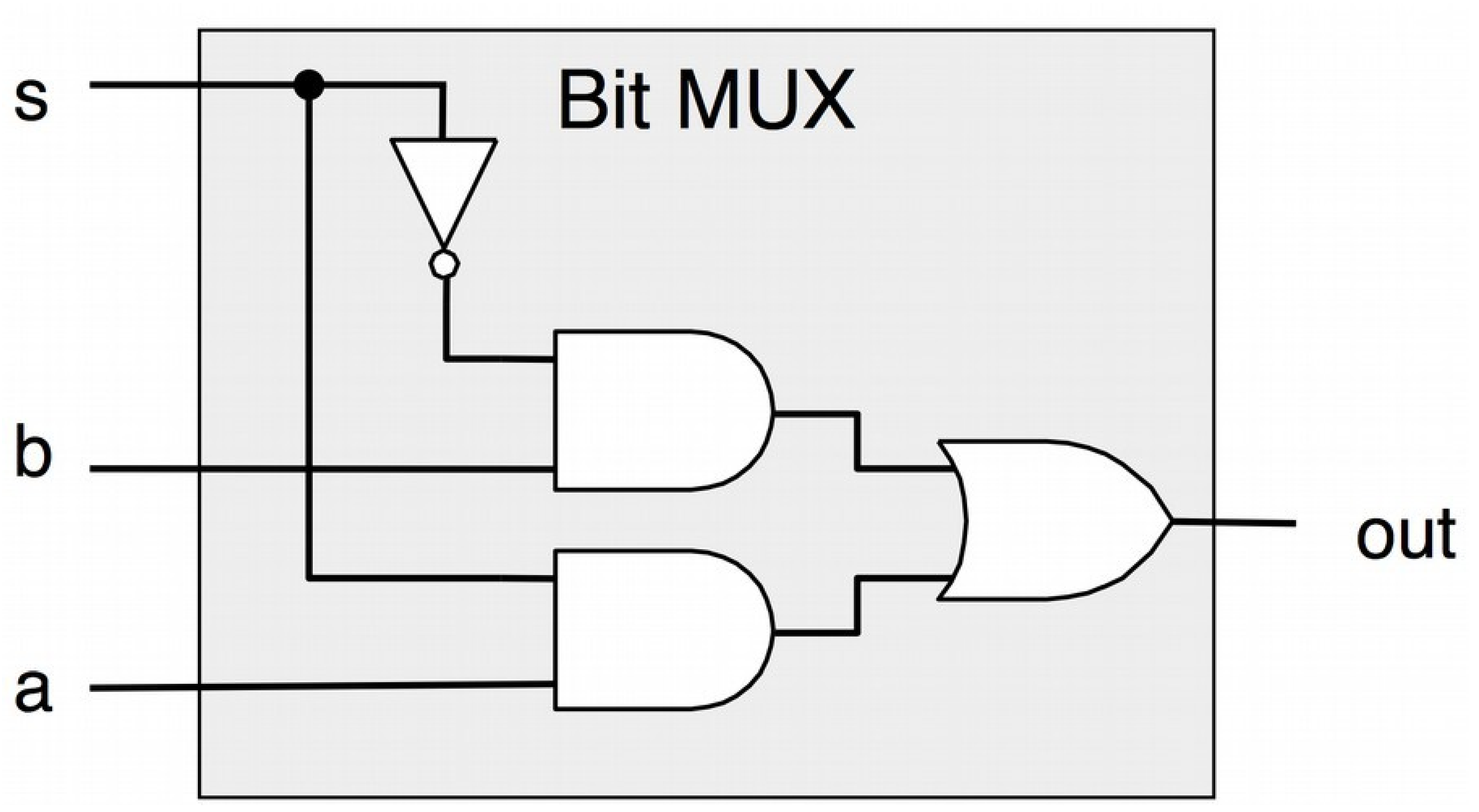
Equality



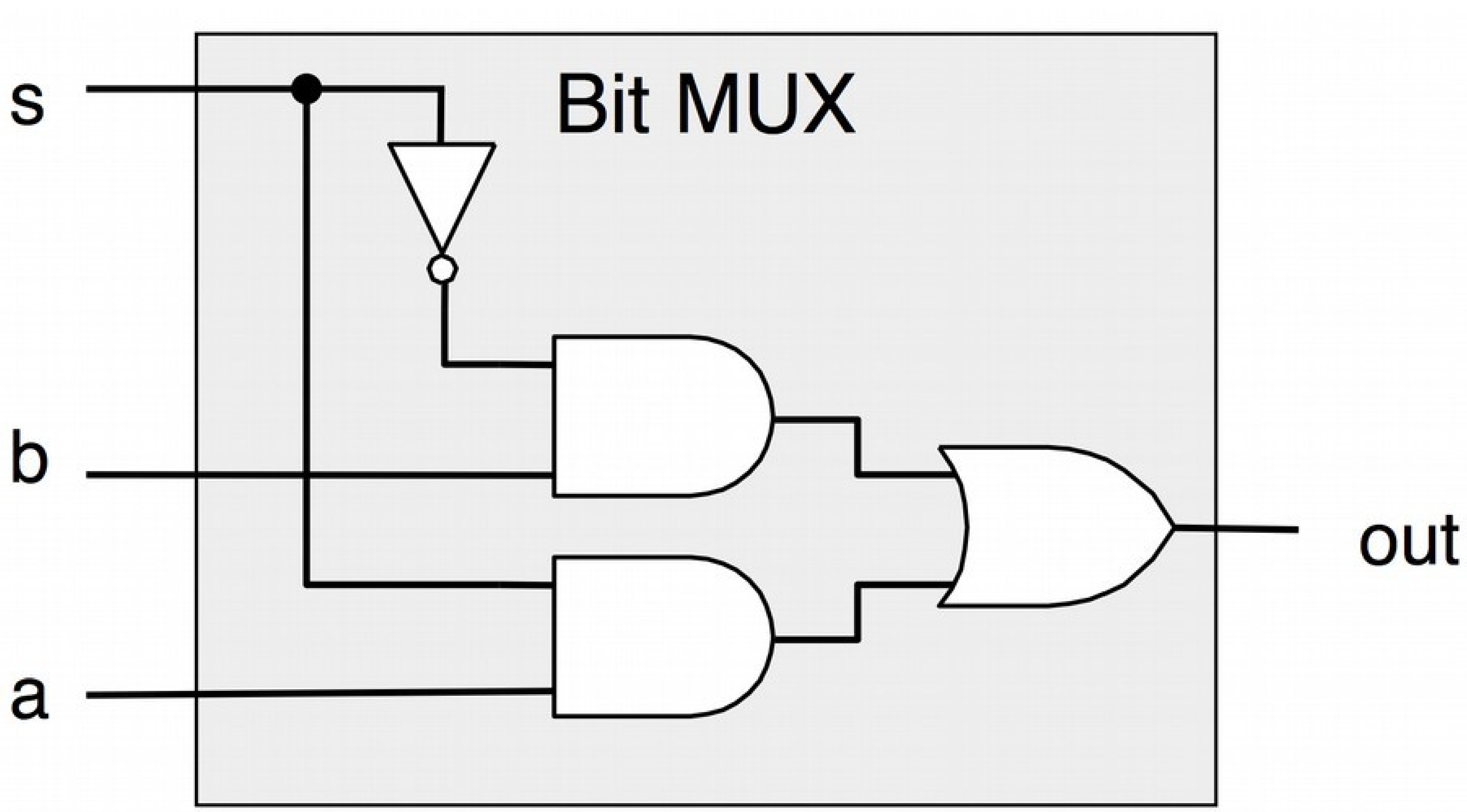
a EQ b = (a & b) | (!a & !b)

EQ(a, b) = OR(AND(a, b), AND(NOT(a), NOT(b)))

Multiplexor ("selector")



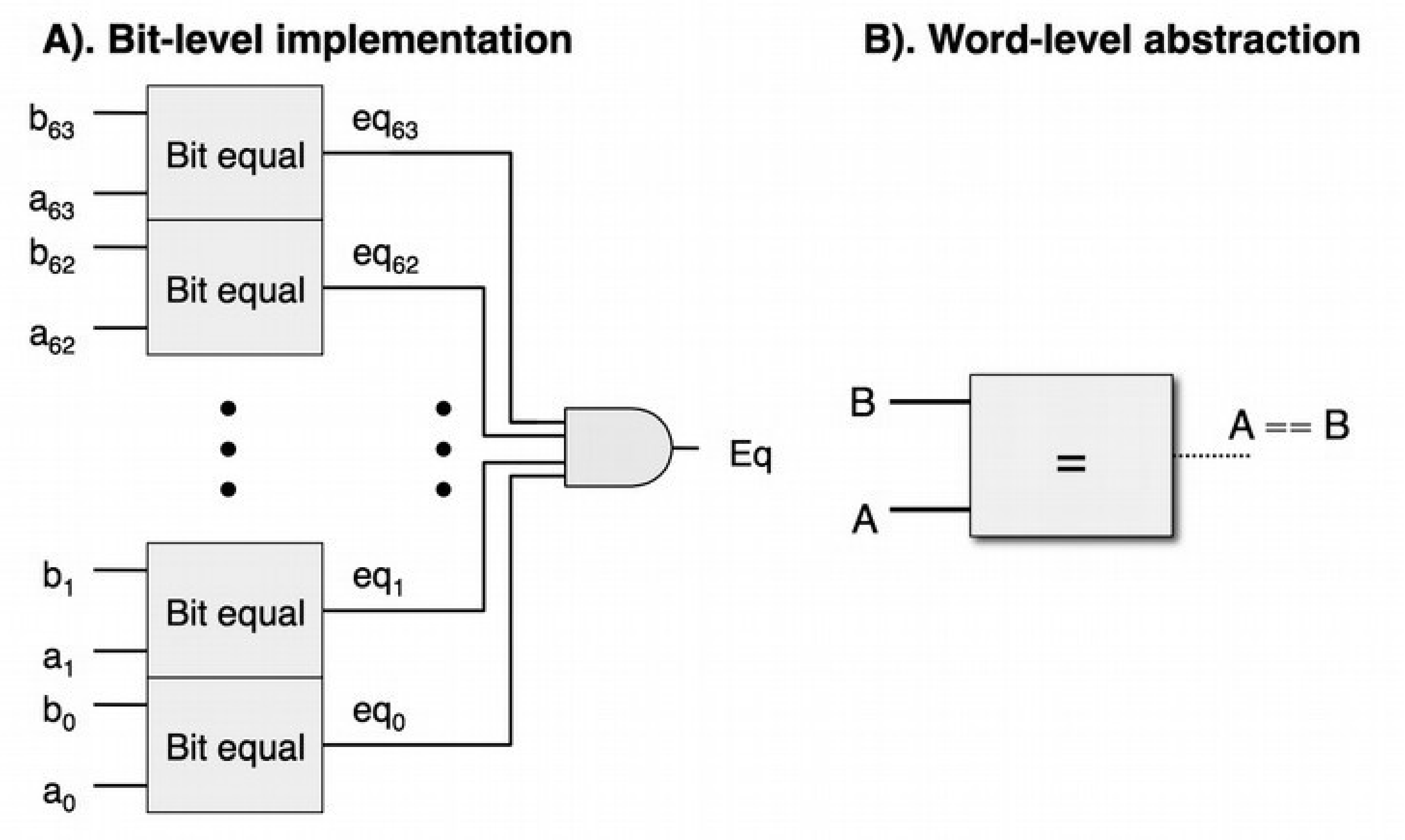
Multiplexor ("selector")



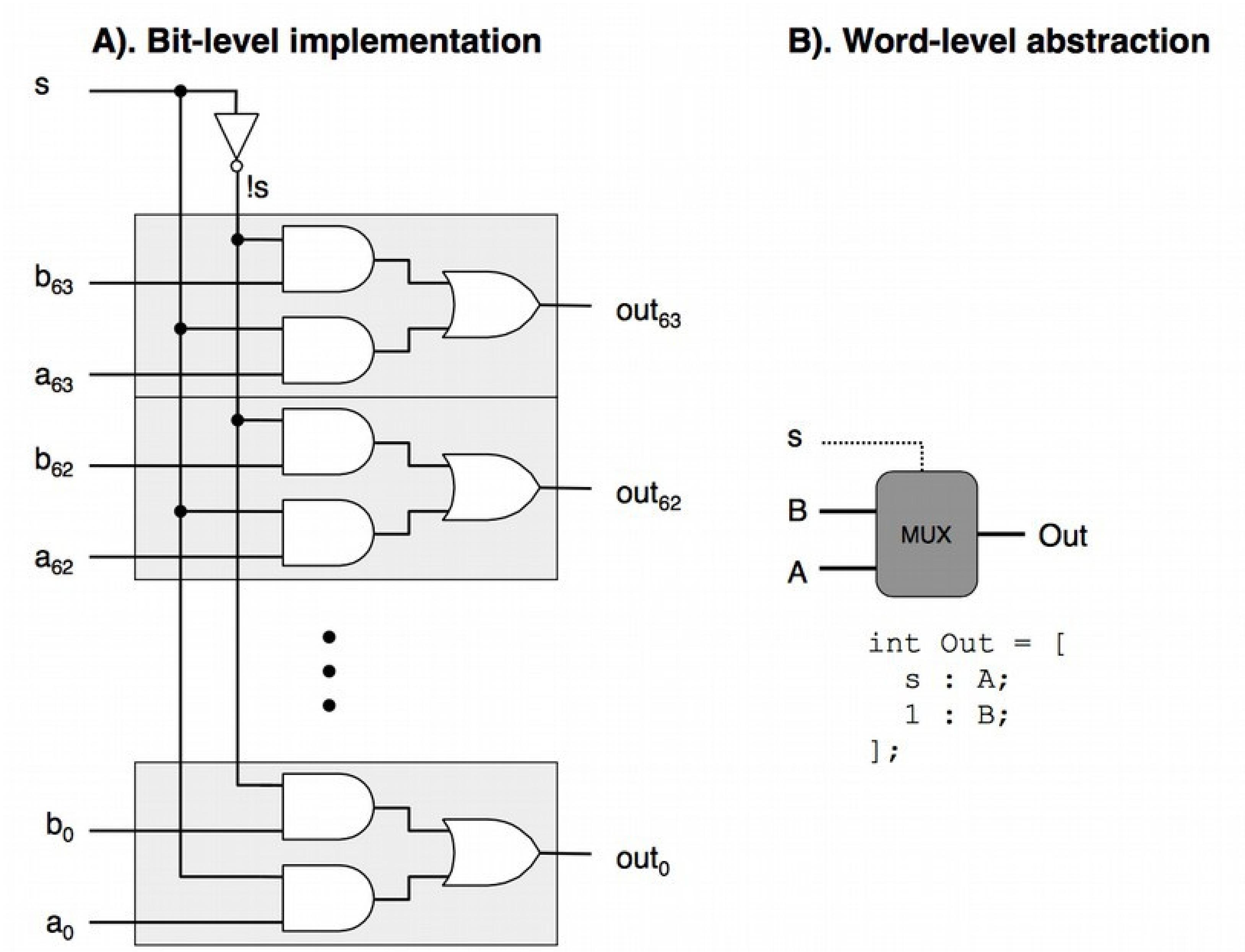
MUX (a, b, s) = (s & a) | (!s & b)

Abstraction

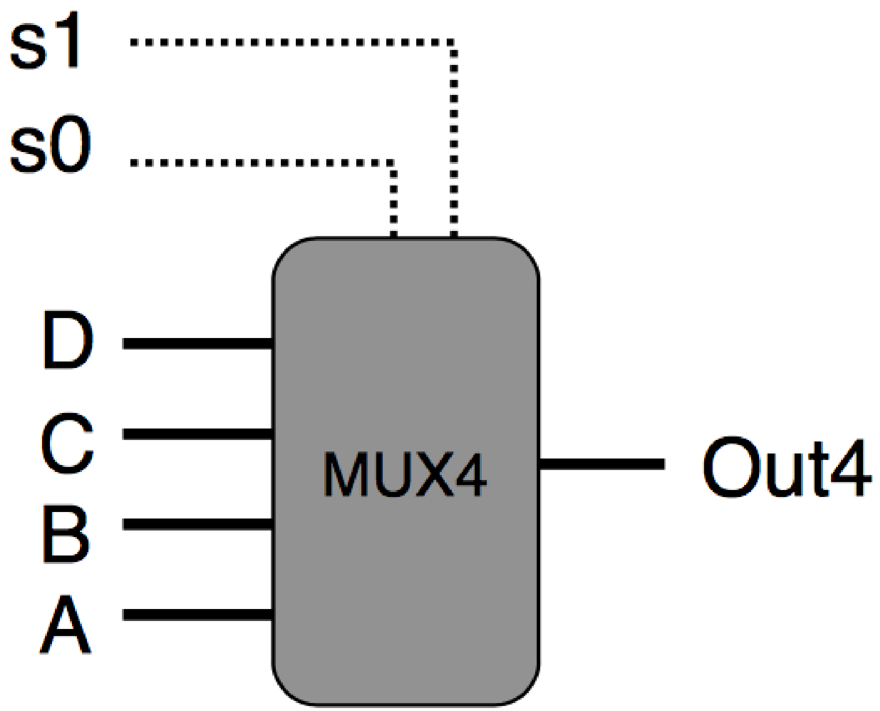
- Name circuits, then use them to build more complex circuits
  - E.g., use bit-level EQ to build a word-level equality circuit:



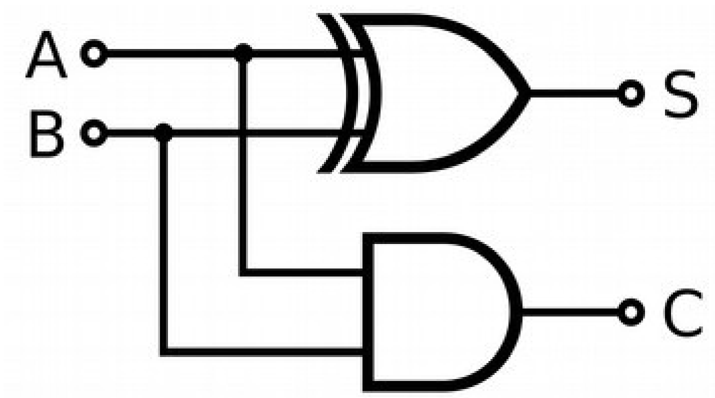
Word-level 2-way multiplexer



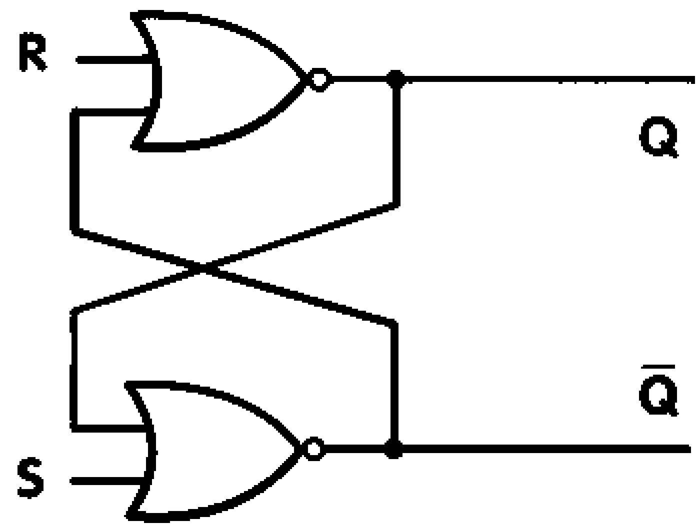
Word-level 4-way multiplexer



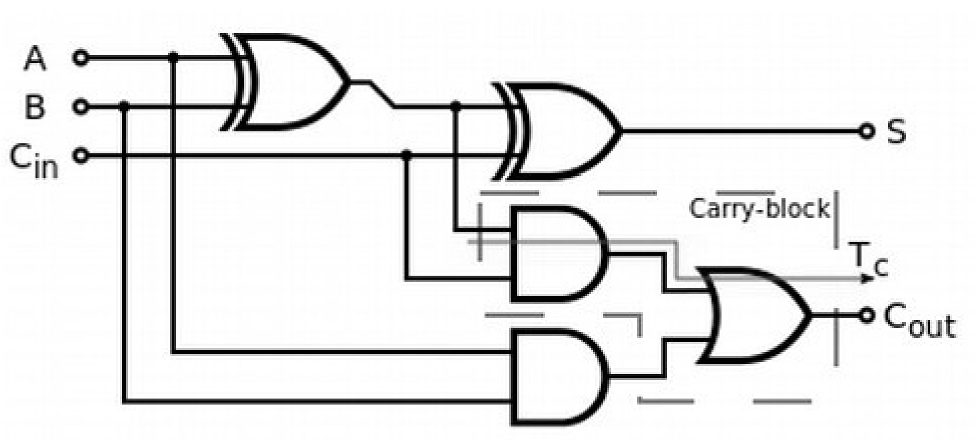
Adders and flip-flops



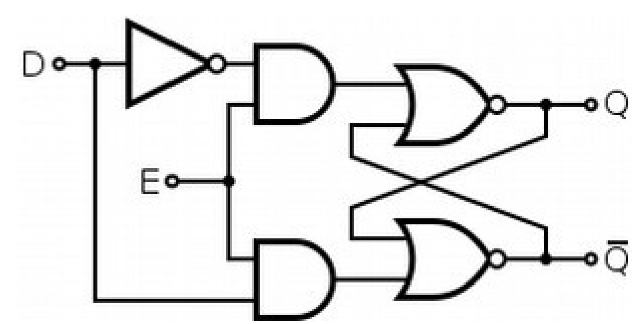
Half Adder



SR Flip-flop



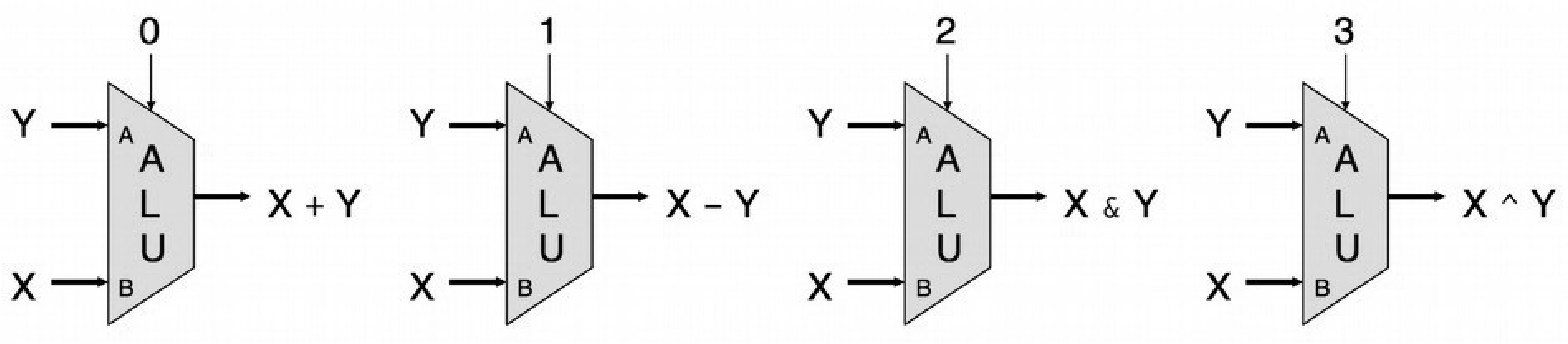
Full Adder



Gated D Flip-flop

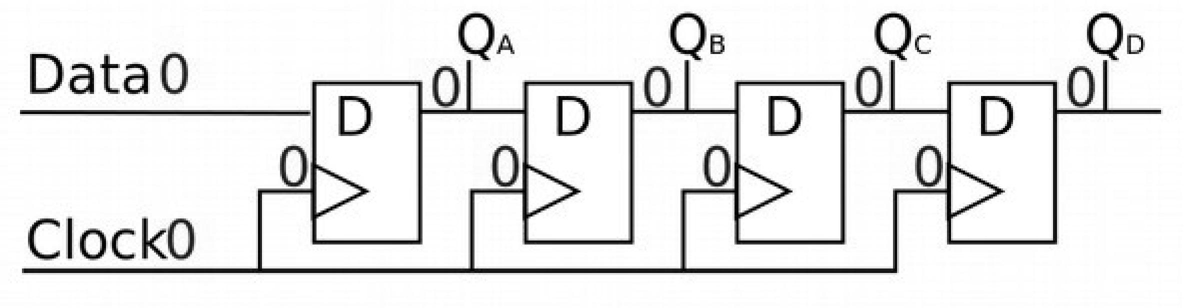
ALUs and memory

- Combine adders and multiplexors to make arithmetic/logic units
- Combine flip-flops to make register files and main memory



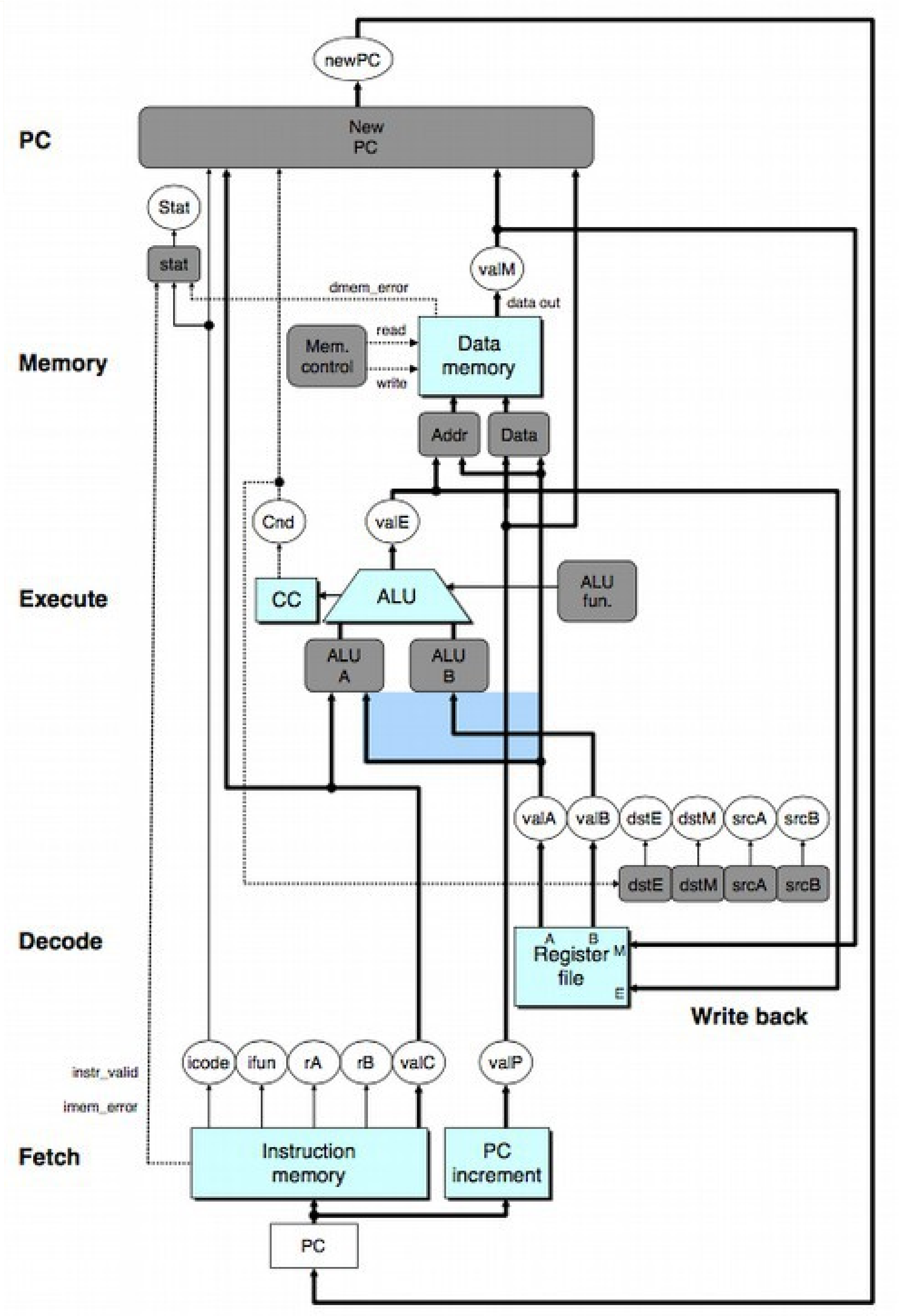
Basic Arithmetic Logic Unit (ALU)

4-bit Register



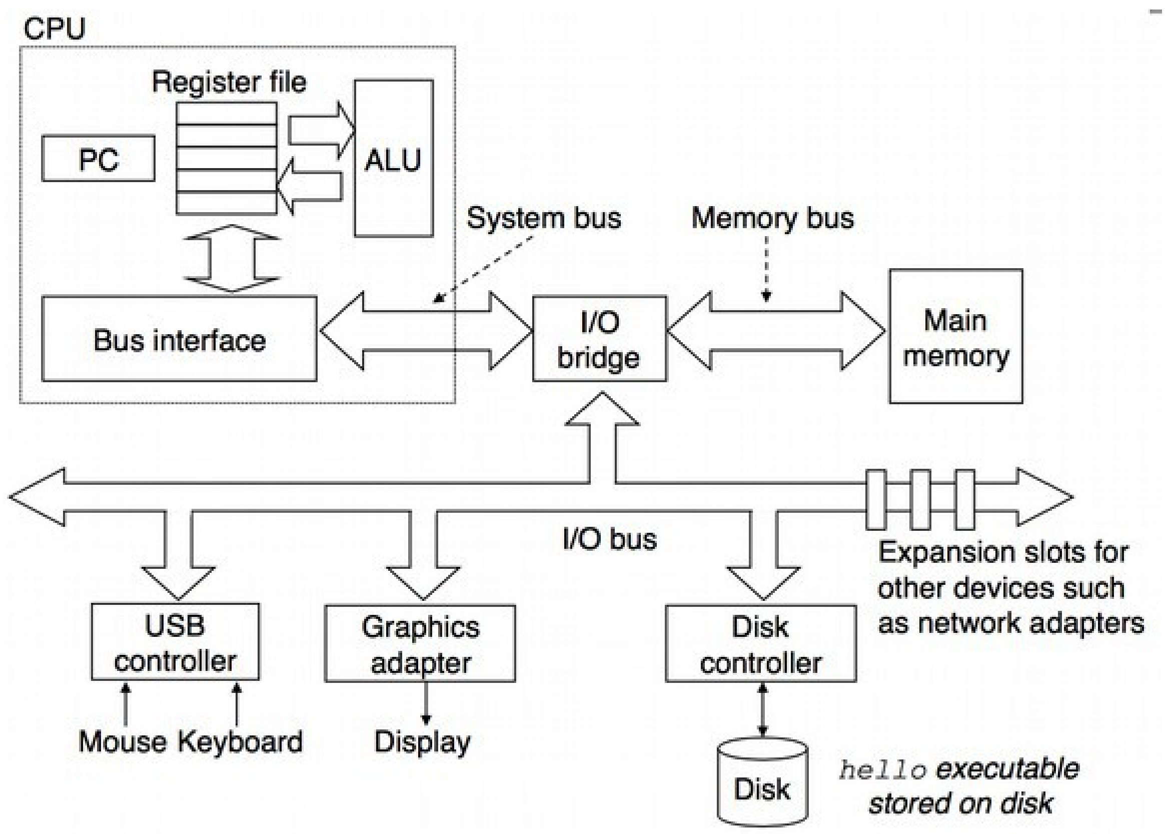
CPUs

- Combine ALU with registers and memory to make CPUs



Computers

- Combine CPU with other electronic components and devices (similarly constructed) communicating via buses to make a computer



Big picture

- Basic systems design approach: exploit abstraction
  - Start with simple components
  - Combine to make more complex components
  - Repeat using the new components as black box “simple components”
- This is true of most areas in systems
  - **CS 261**: transistors → gates → circuits → adders/flip-flops → ALUs/registers → CPUs/memory → computers
  - **CS 261**: machine code → assembly → C code → Java/Python code
  - **CS 361/470**: threads → processes → nodes → networks/clusters
  - **CS 432**: scanner → parser → analyzer → code generator → optimizer
  - **CS 450**: files + processes + I/O → kernel → operating system

Course status

- We’ve hit the bottom
  - Or at least as far down as we’re going to go (logic gates)—from here we go back up!
- Next week
  - Combinational circuits
  - Sequential circuits
  - CPU architecture

Suggestion: download **Logisim** (already installed on lap machines) and play around with some circuits!