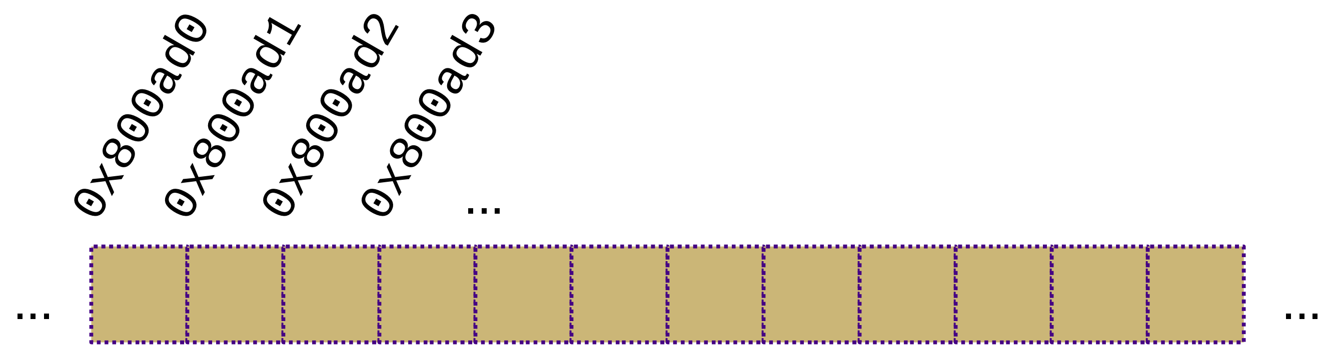
CS240
Fall 2014

Mike Lam, Professor

Dynamic Arrays

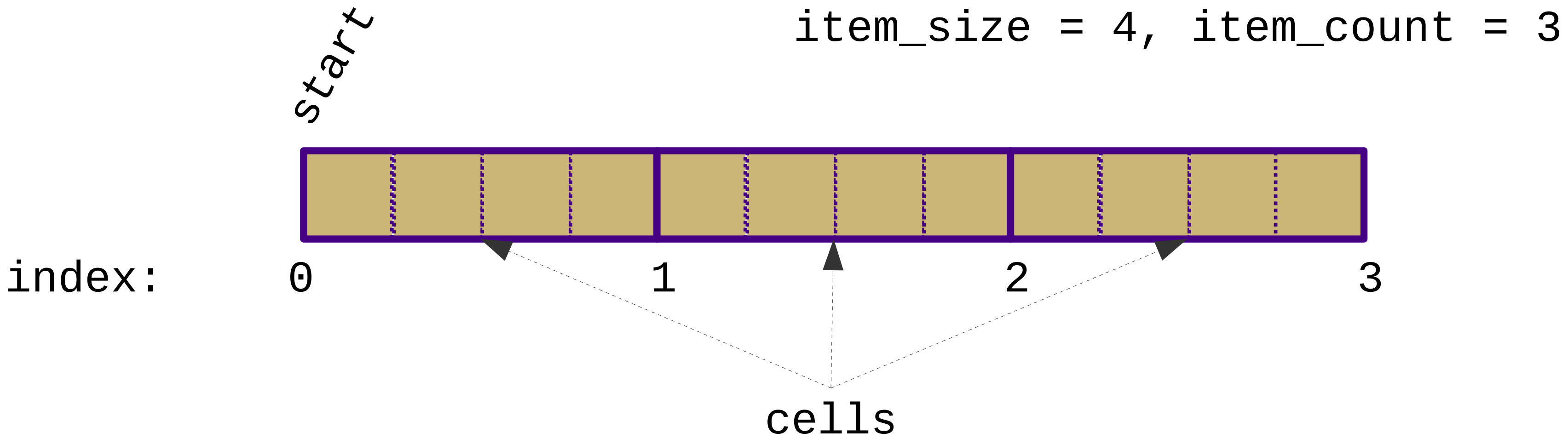
Computer Memory

- Lowest level: sequence of bytes
- Each byte has a 32-bit or 64-bit address
- Every byte is equally easy to access
  - "Random access" memory



Arrays

- Finite sequence of uniformly-sized segments
  - Starting address, item size (in bytes), item count (fixed)
- Each location is a **cell** located at a zero-based **index** offset from the start
  - Address of cell `i` is `start+(i*item_size)`



Compact Arrays

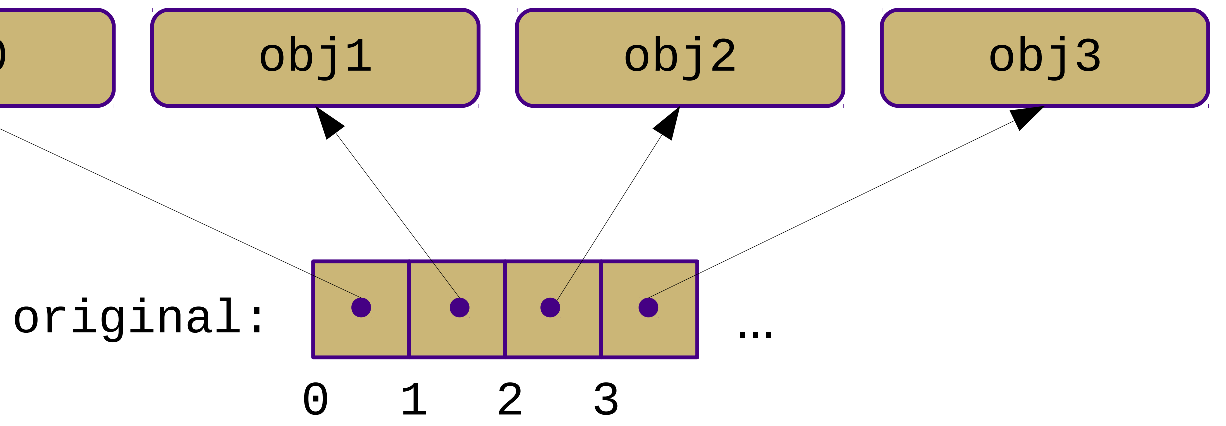
- In some languages, compact byte arrays are part of the language
  - Stack (C/C++)
    - `int my_array[n]`
  - Heap (C/C++)
    - `my_array = (int*)malloc(n*sizeof(int))`
  - Heap (C++/Java)
    - `my_array = new int[n]`

Compact Arrays

- In Python, compact byte arrays of built-in types are supported by the array module
  - `from array import array`
  - `my_array = array('i', [0]*n)`
- However, Python lists are not compact arrays!
  - Referential
  - Not fixed-length

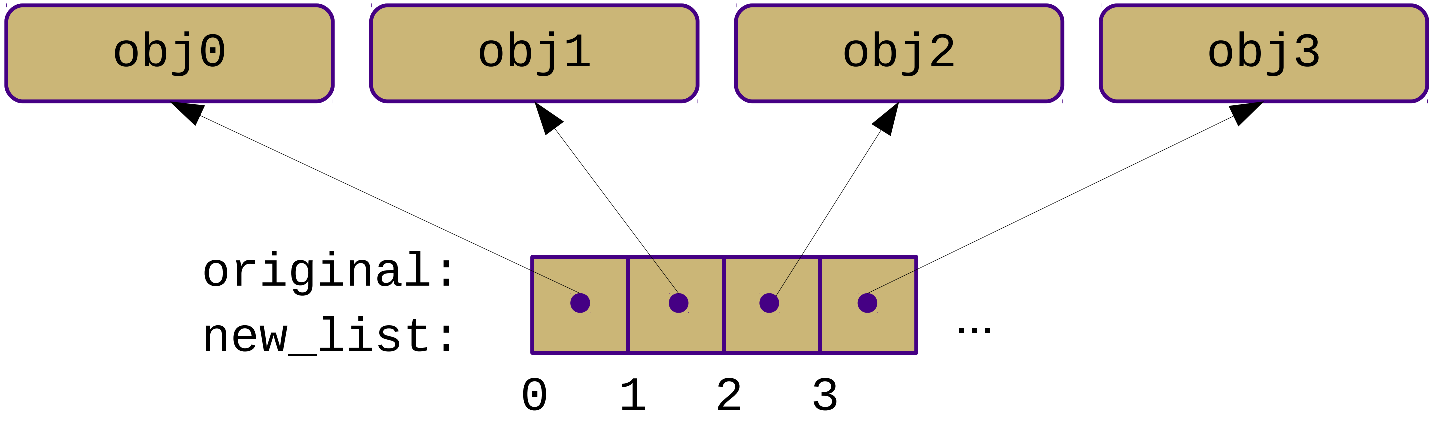
Referential Array

- Array of references
- Each cell contains a 32 or 64 bit pointer to actual objects
- This is how Python implements lists



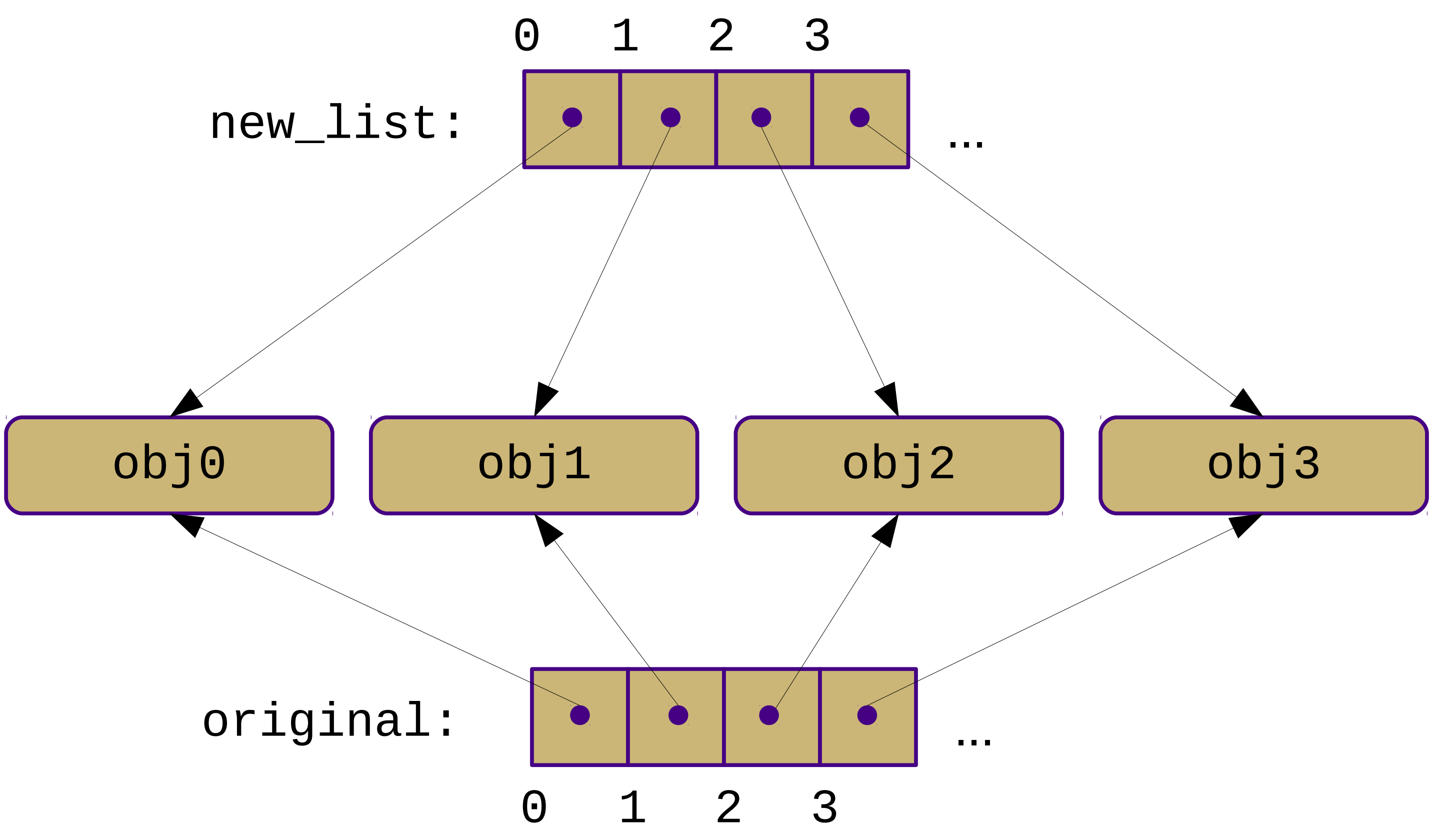
Shallow vs. Deep Copy

- Alias: `new_list = original`



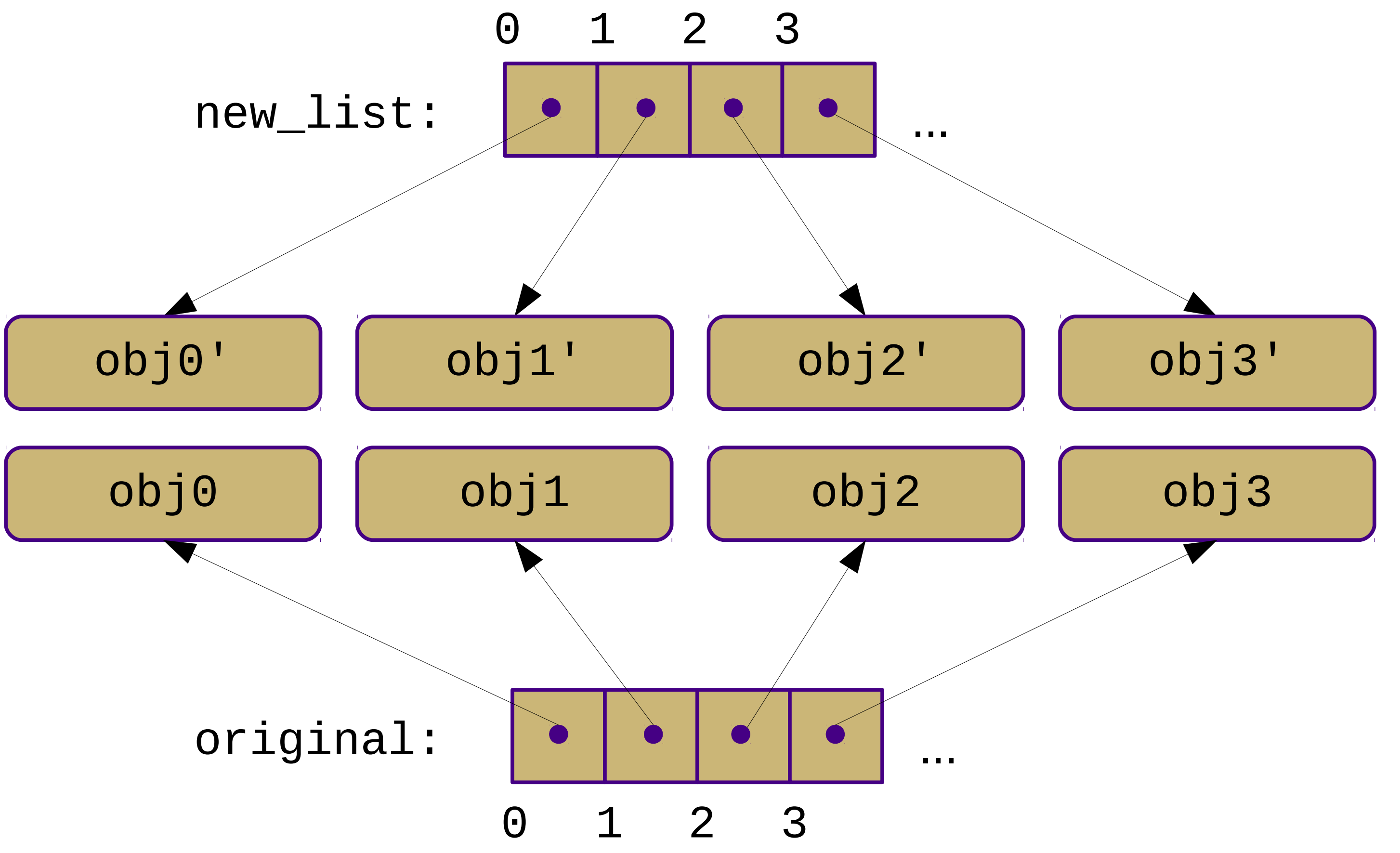
Shallow vs. Deep Copy

- Shallow copy: `new_list = list(original)`



Shallow vs. Deep Copy

- Deep copy: `new_list = copy.deepcopy(original)`



Python Lists

- How does the append operation work in Python?
  - Standard arrays are fixed-length
  - Python uses "dynamic arrays"

Dynamic Arrays

- Goal: Add items to an array
- Issue: Arrays are fixed-length

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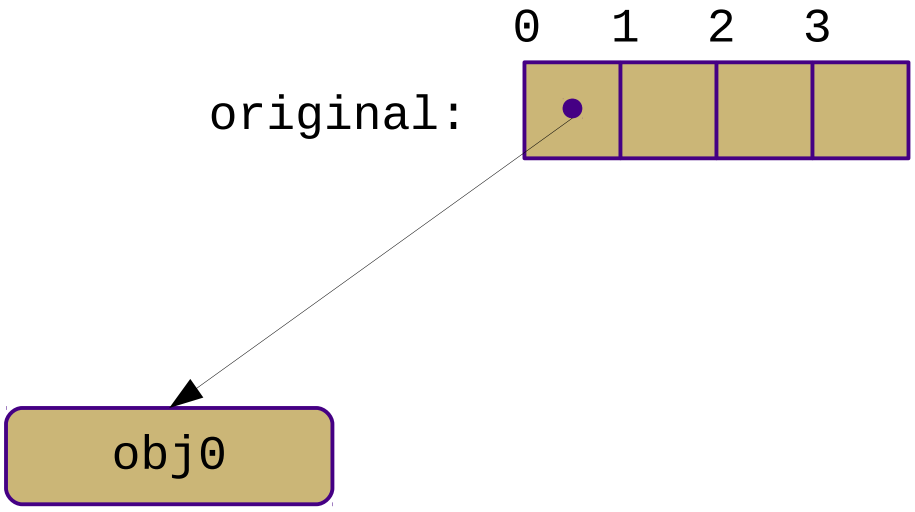
  - Problem: no guarantee that we can do this!

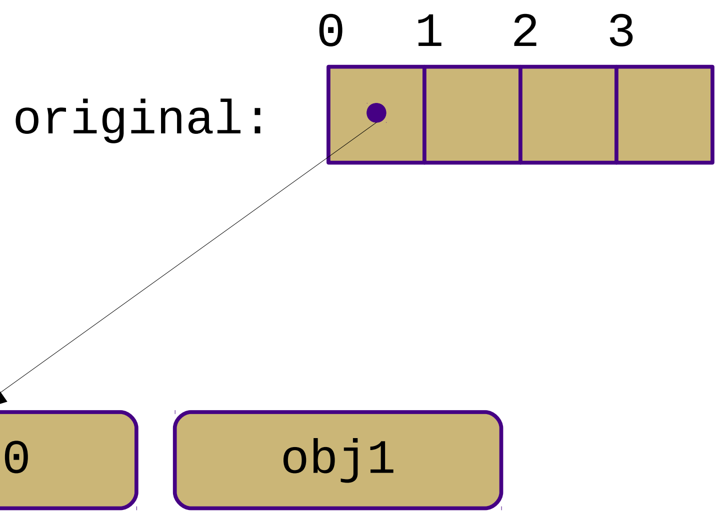
Dynamic Arrays

- Goal: Add items to an array
- Issue: Arrays are fixed-length
- Naive solution: Resize the array
  - Problem: no guarantee that we can do this!
- More robust solution: Dynamic arrays
  - Allocate more space than currently needed
  - Re-allocate and copy when the original size is exceeded

Dynamic Arrays



Dynamic Arrays



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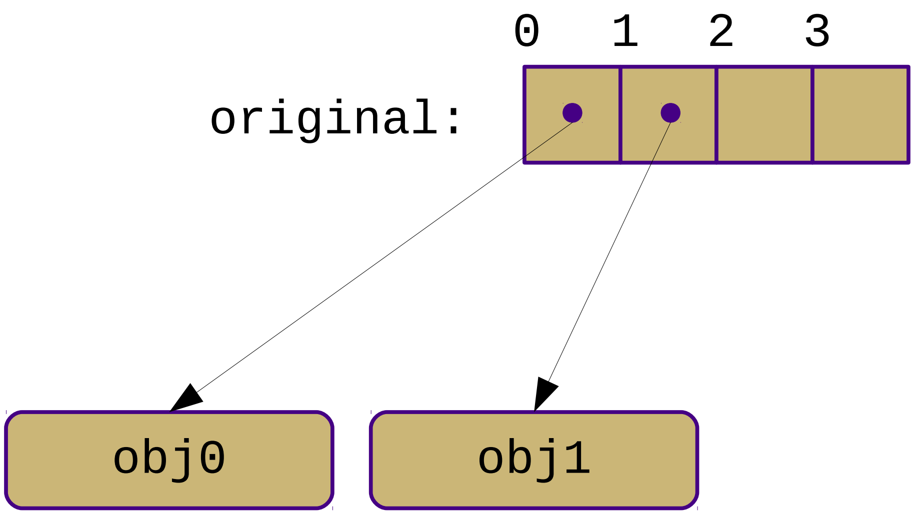
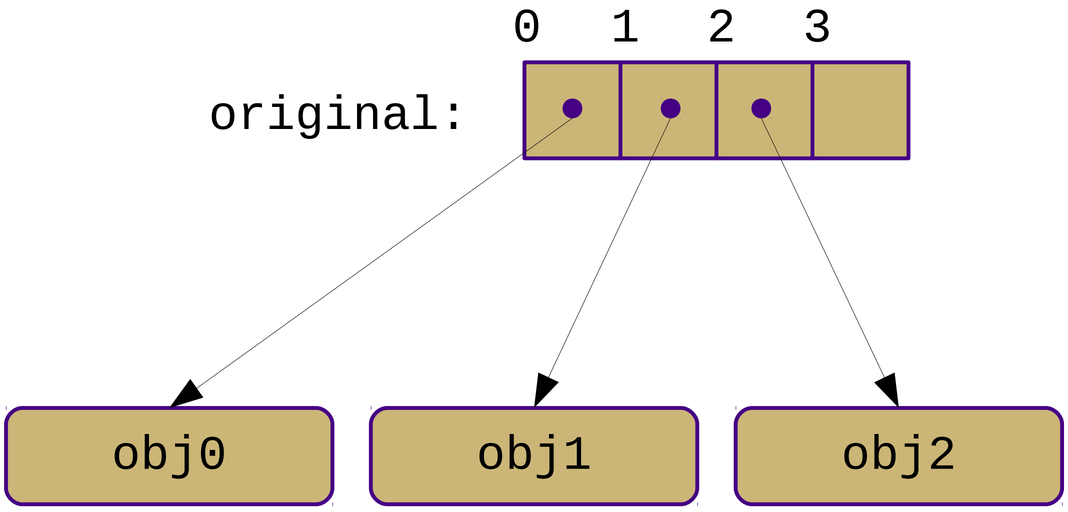


Dynamic Arrays



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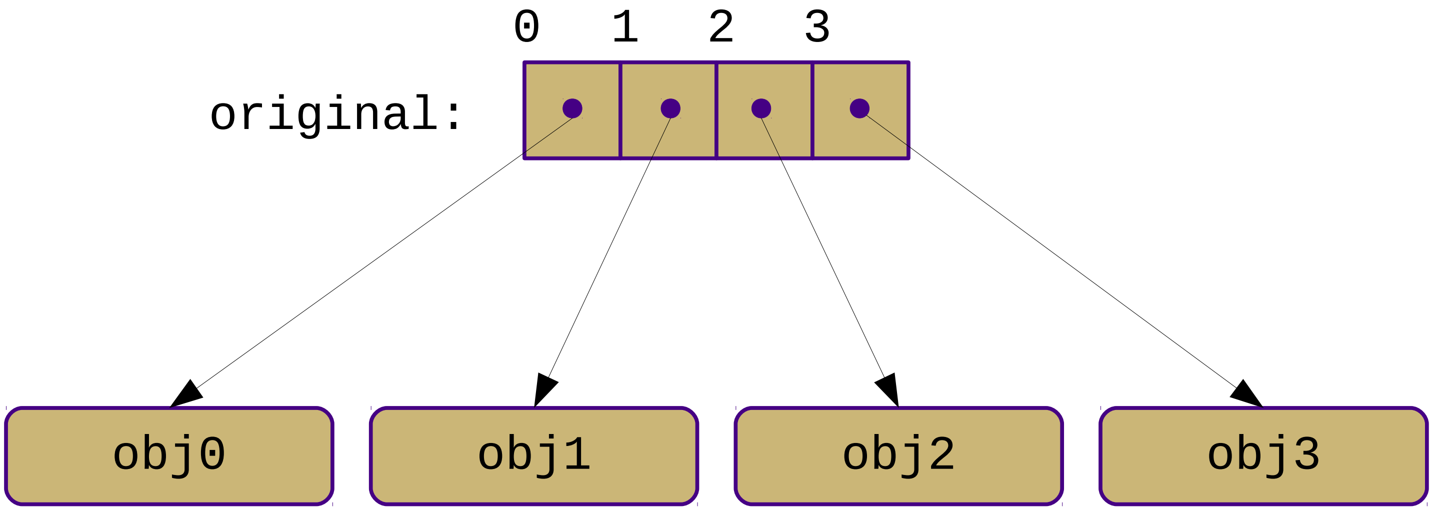

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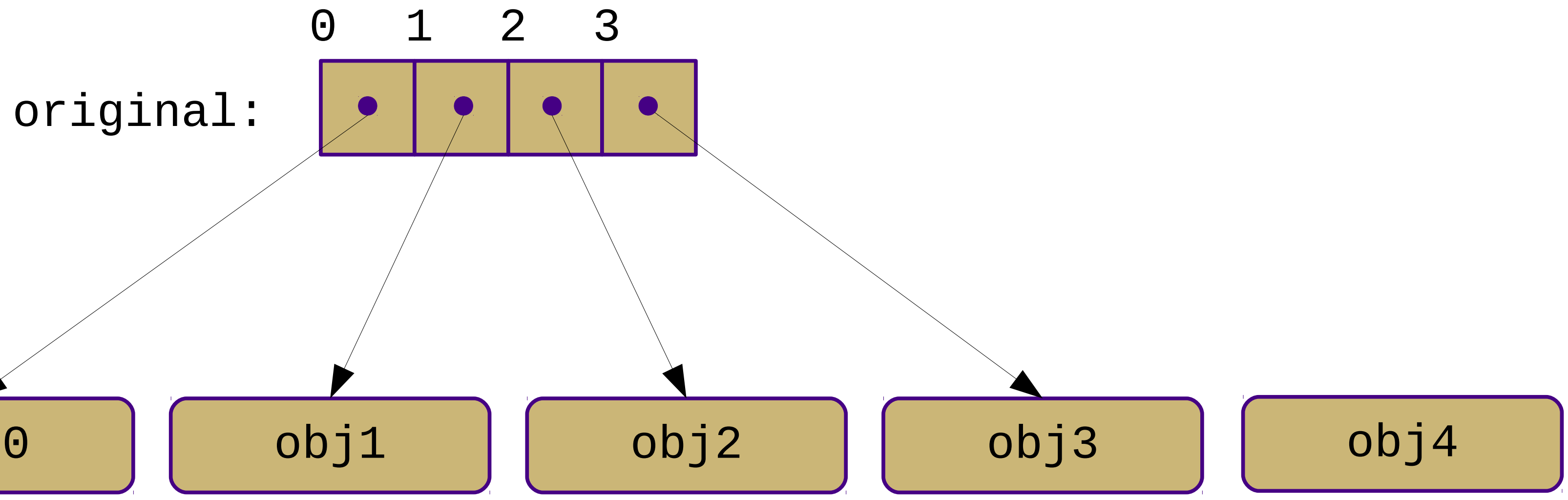

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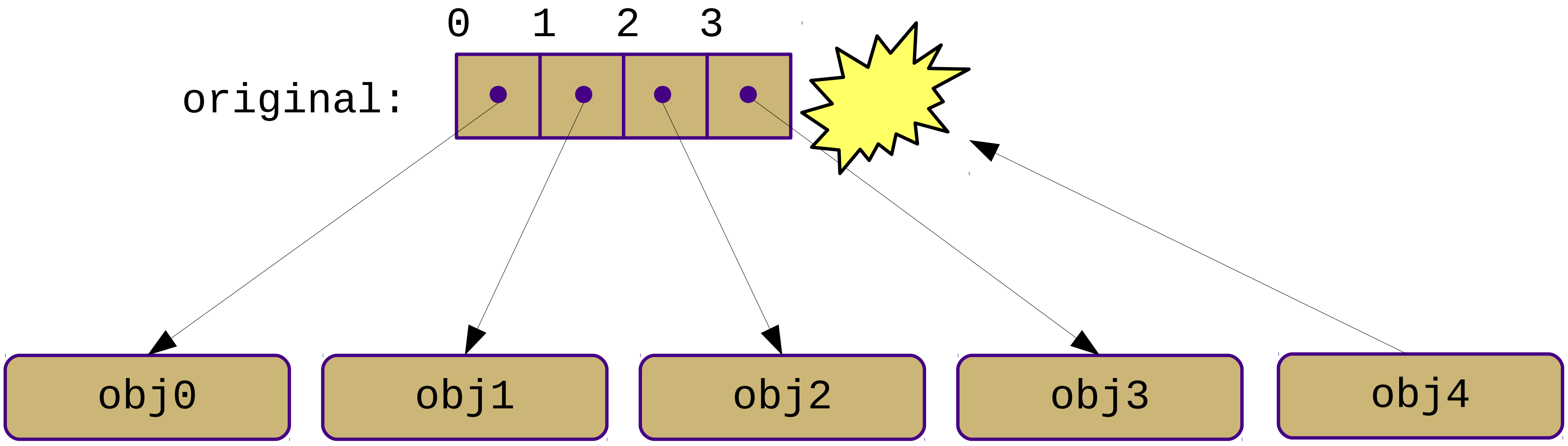

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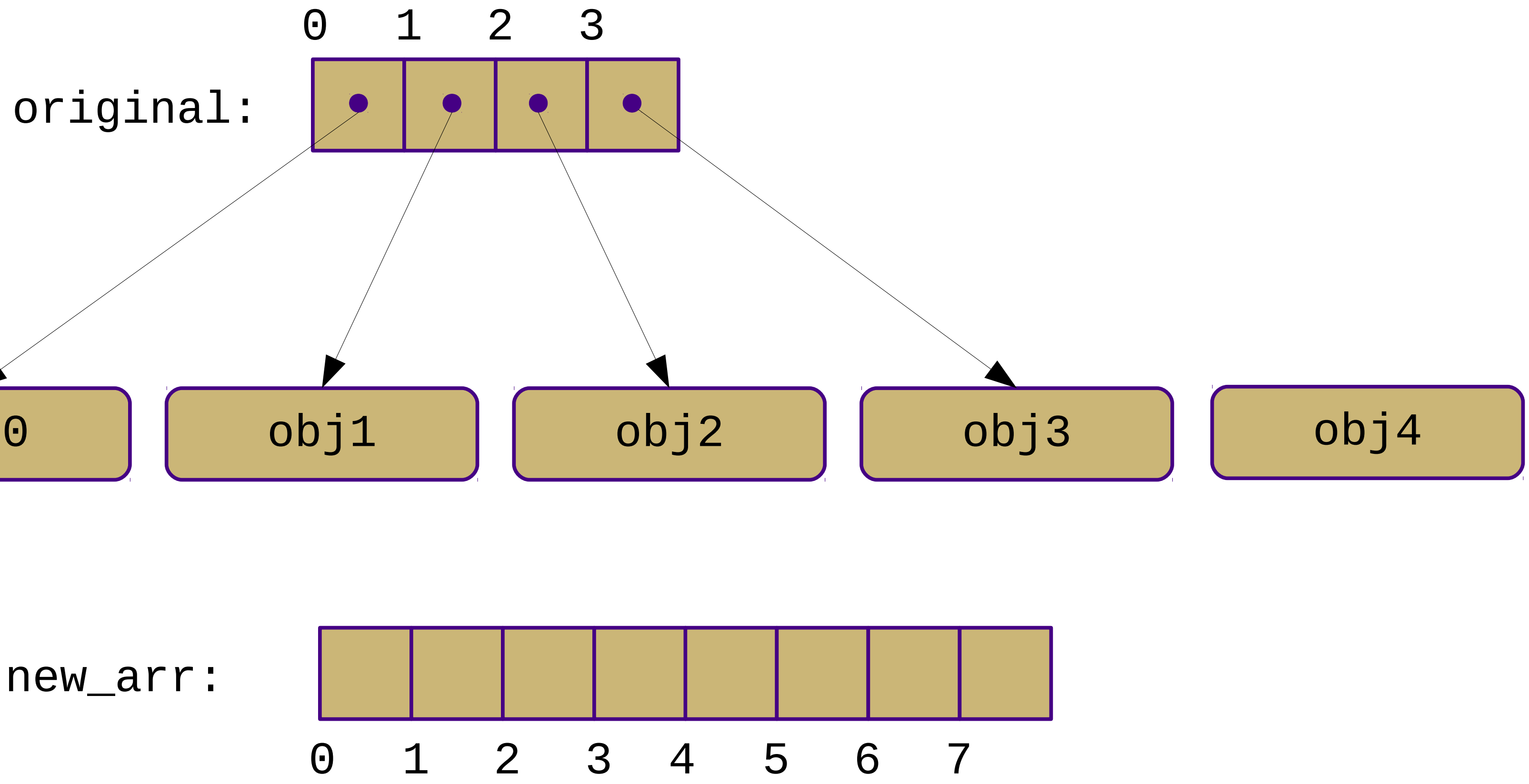

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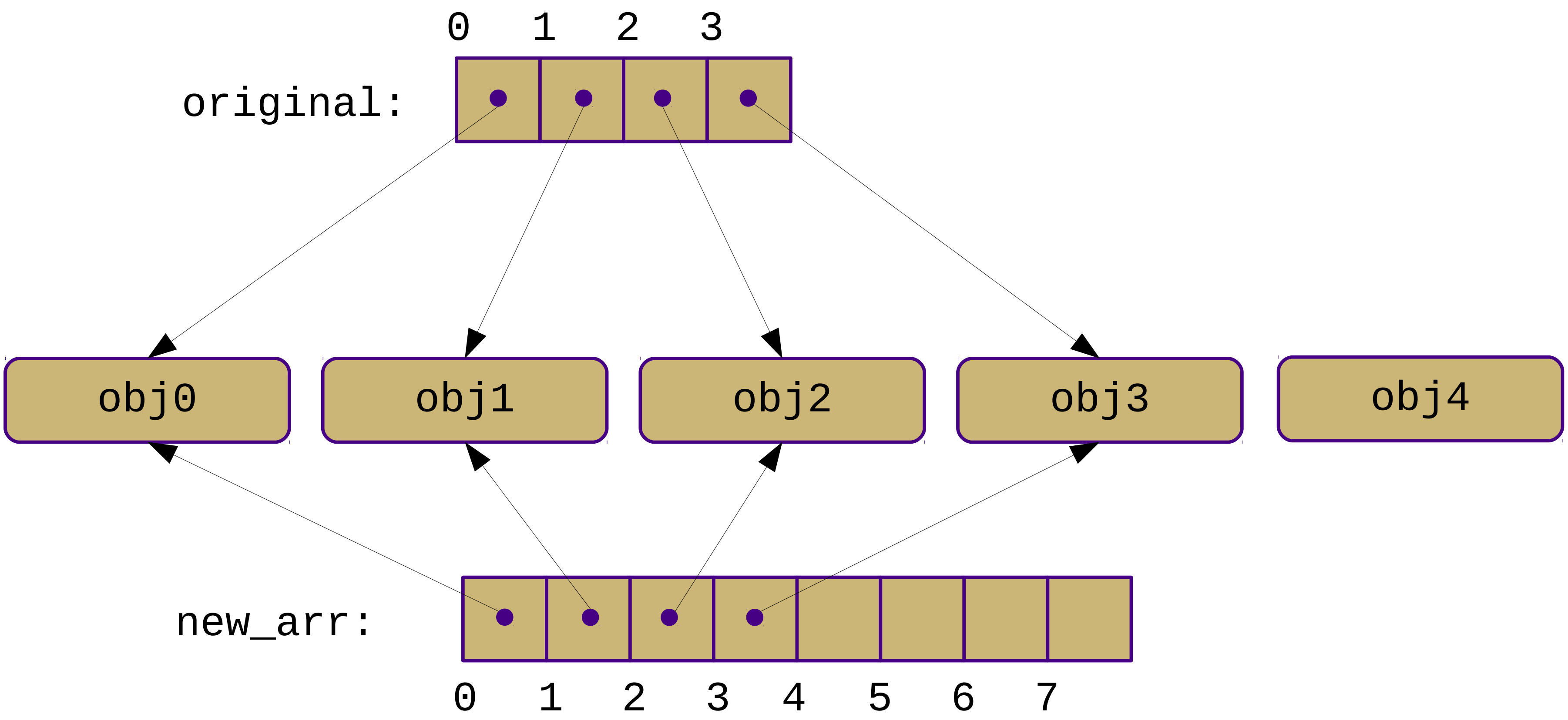

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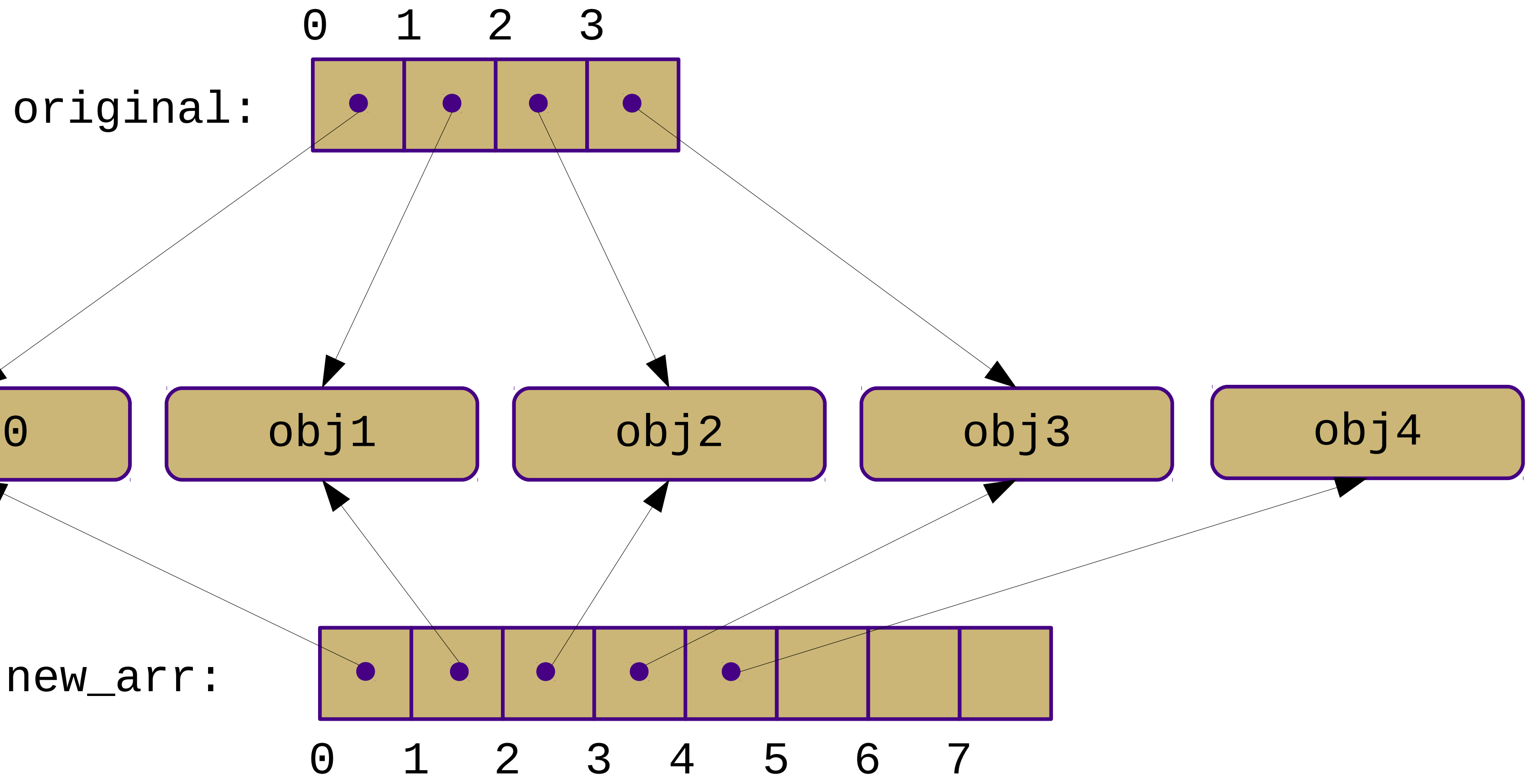
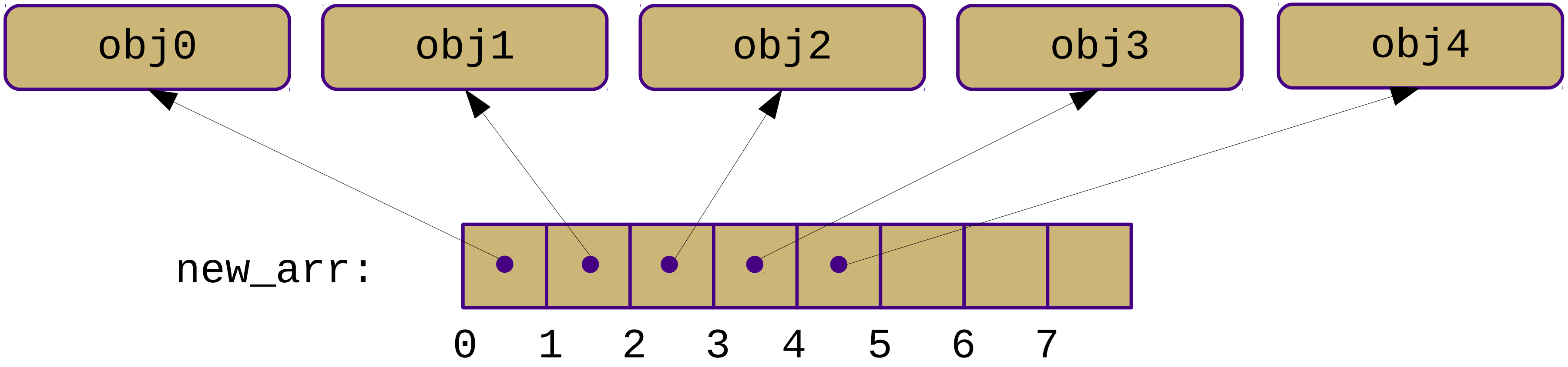


Dynamic Arrays



Dynamic Arrays

- State information:
  - **n**: current element count
  - **cap**: current maximum element count
  - **arr**: array reference
- Invariant: **cap** >= **n**

Dynamic Arrays

- How big should we initialize new arrays?
  - For now let's make it big enough for a single element
- How much extra space should we allocate when we need to resize it?
  - For now, let's assume we double the size

Dynamic Arrays

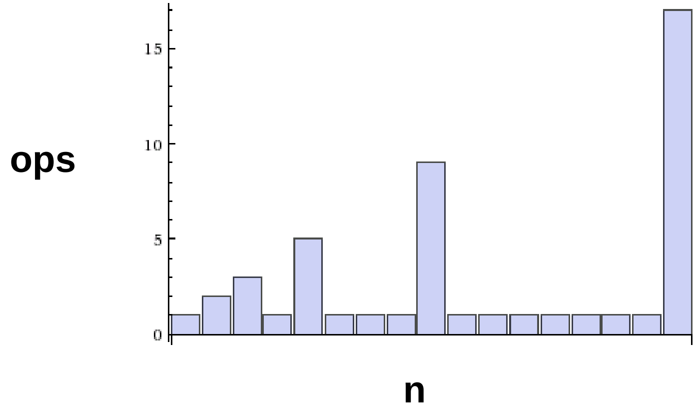
- See code example
  - (simpler than book example)
  - (uses built-in lists instead of ctypes)

Dynamic Arrays

- Big-O analysis
  - Create empty array: O(1)
  - Access element: O(1)
  - Modify element: O(1)
  - Get length: O(1)
  - Append element: ???
    - Let's measure cost in "copy operations"

Dynamic Arrays

- Big-O analysis
  - Create empty array: O(1)
  - Access element: O(1)
  - Modify element: O(1)
  - Get length: O(1)
  - Append element:
    - If **cap** > **n**: O(1)
    - If **cap** == **n**: O(n)



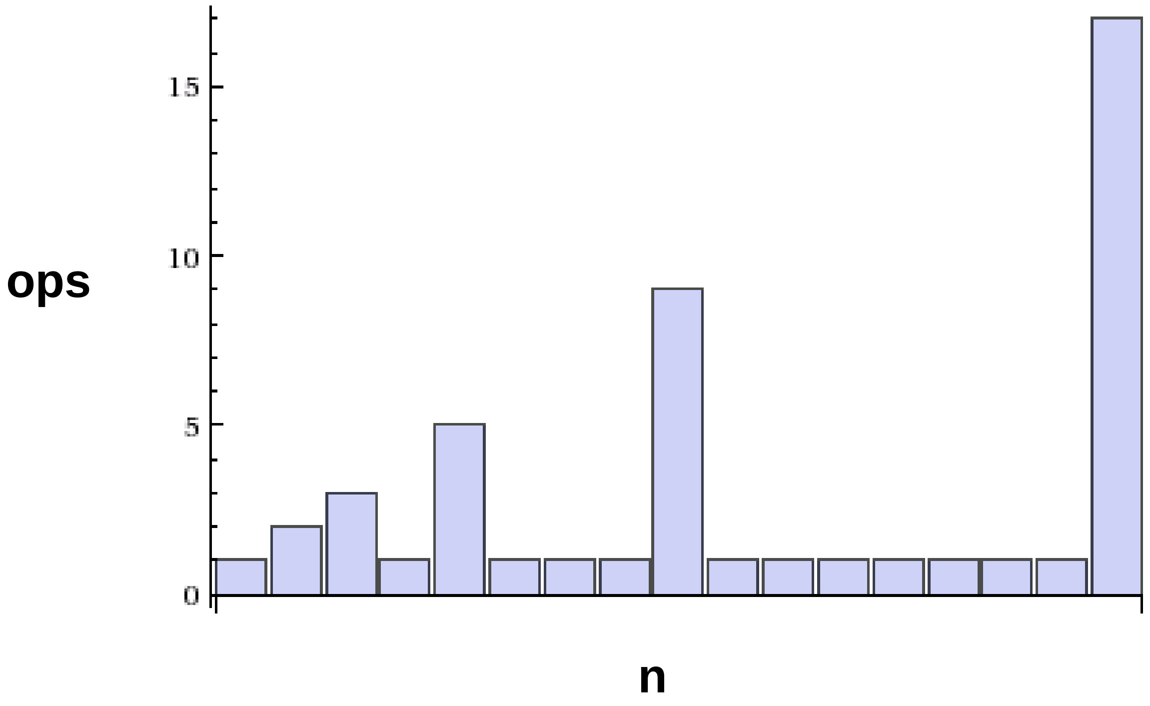
n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
cap	1	2	4	4	8	8	8	8	16	16	16	16	16	16	16	16	32
ops	1	2	3	1	5	1	1	1	9	1	1	1	1	1	1	1	17

Dynamic Arrays

- Can we argue that the *average* cost of the append operation is O(1), despite its occasional O(n) cost?
- Yes! Use *amortized* analysis
- Basic idea: charge algorithm $$ to perform operations
  - Overcharge for some (inexpensive) operations
  - Use saved $$ to pay for expensive operations
  - Show that the total $$ spent is O(n) for *n* operations
  - Thus, each operation can be considered O(1)

Amortized Analysis

- Intuition: Cost of rare expensive operations grows inversely proportionally to frequency



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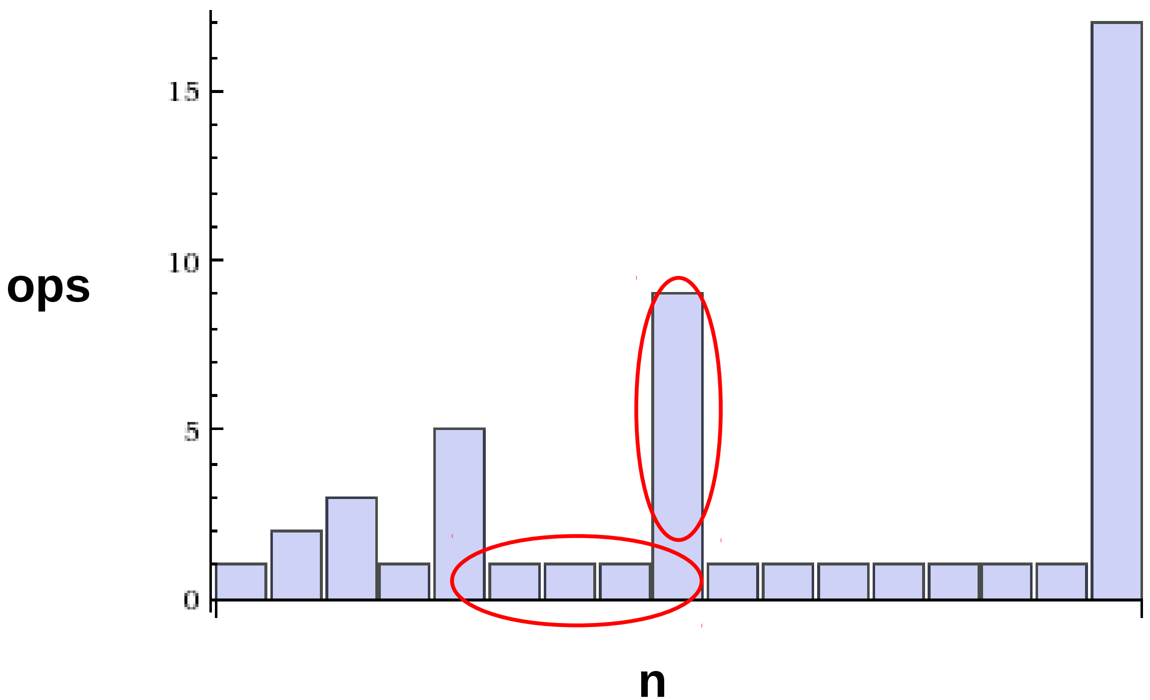

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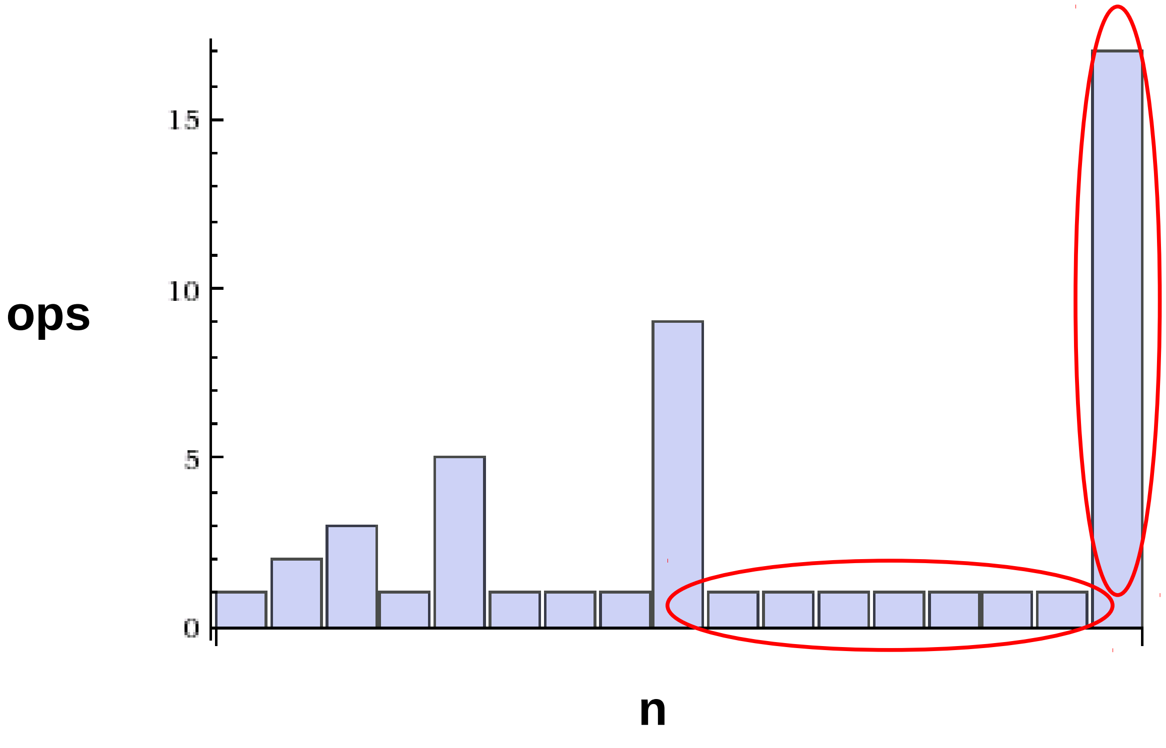
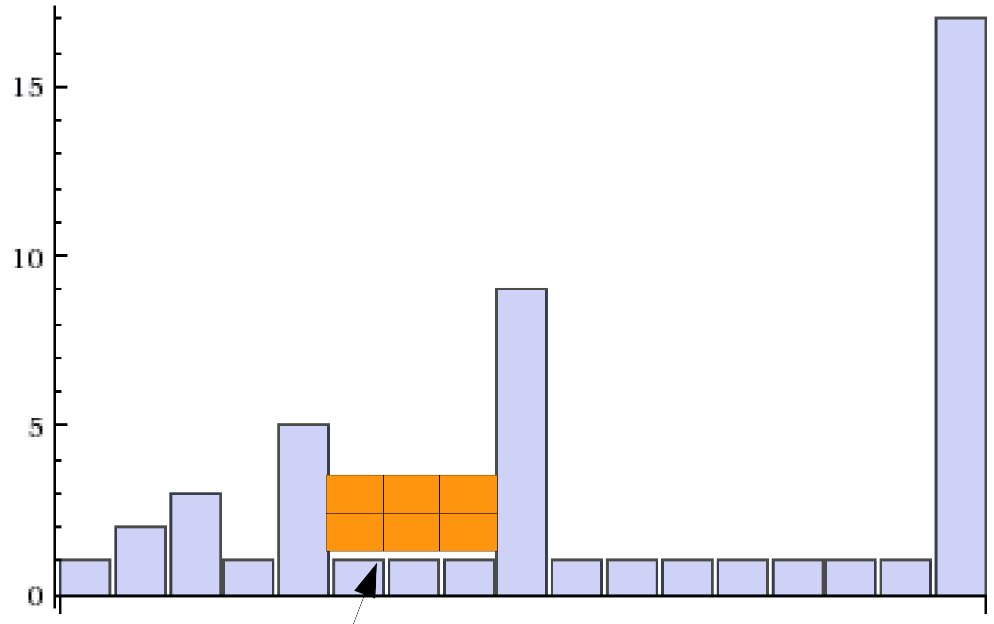


Amortized Analysis

- Idea: Charge extra for O(1) insertions to “save up” and “pay for” the O(n) insertions



Charge extra here

Amortized Analysis

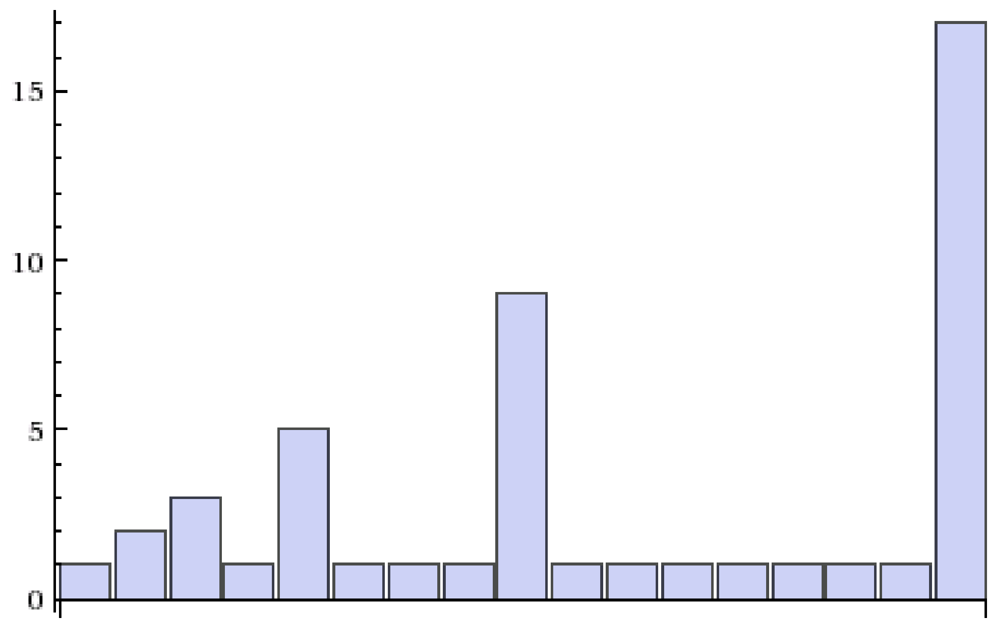
- Idea: Charge extra for O(1) insertions to “save up” and “pay for” the O(n) insertions

Amortized Analysis

- How much extra do we charge?
  - Let's try charging 1 extra operation
  - Total of 2 operations per append

Amortized Analysis

- How much extra do we charge?



Amortized Analysis

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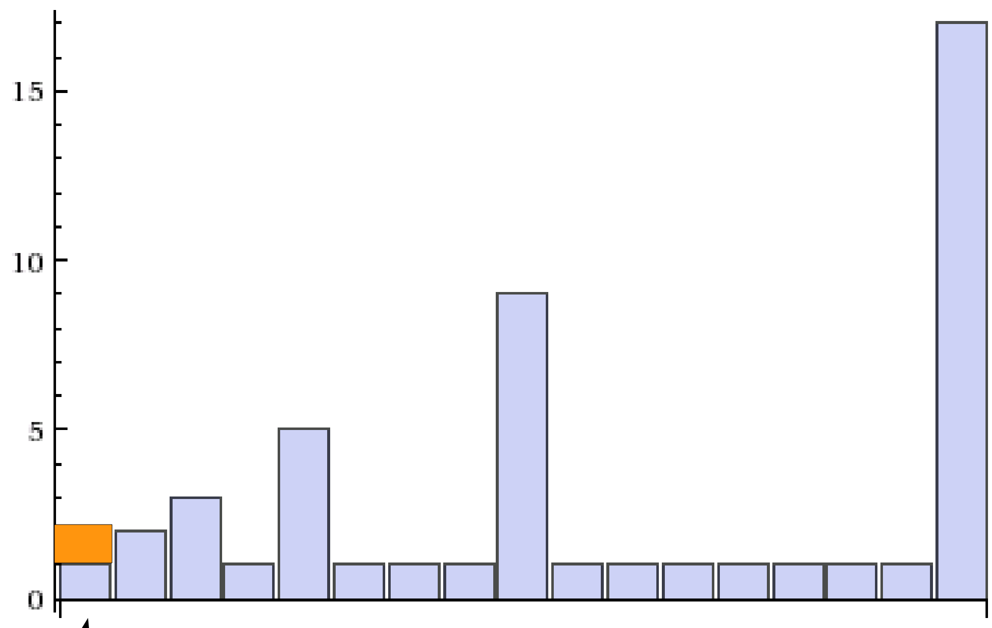
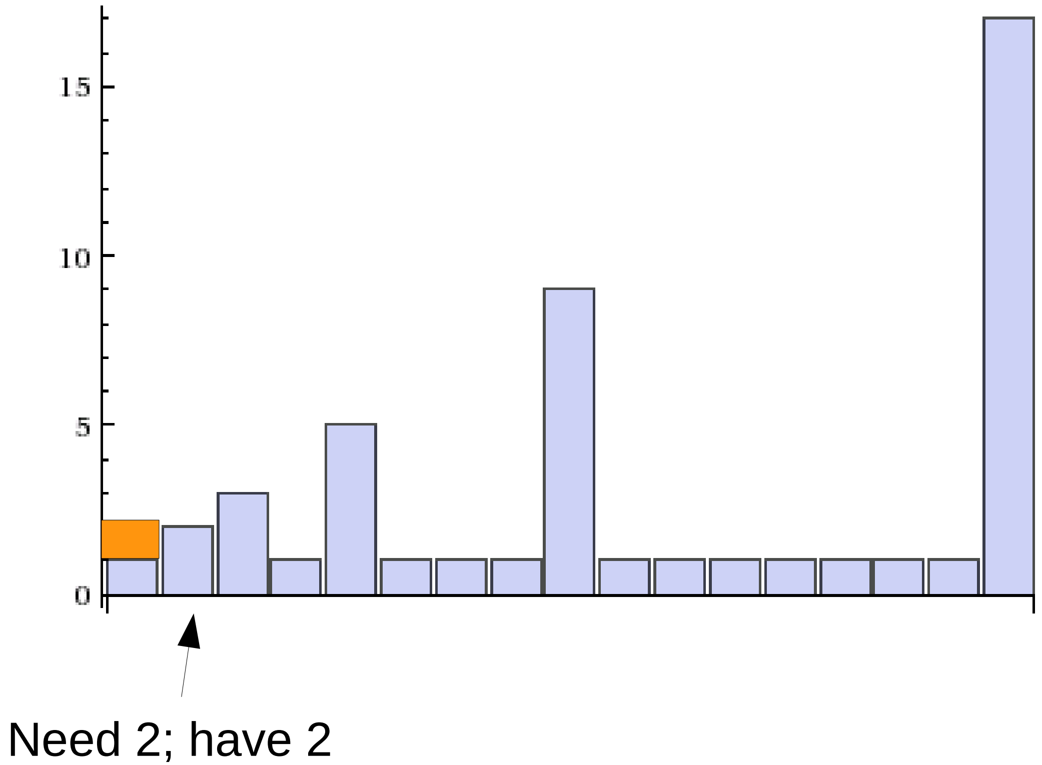


Need 1; have 2; save one!

Amortized Analysis

- How much extra do we charge?



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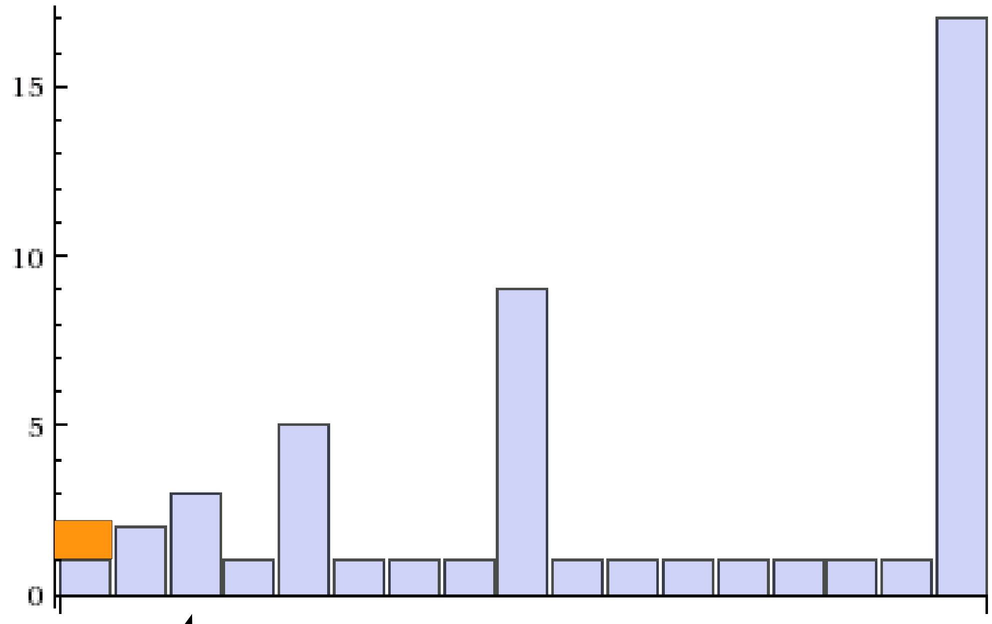
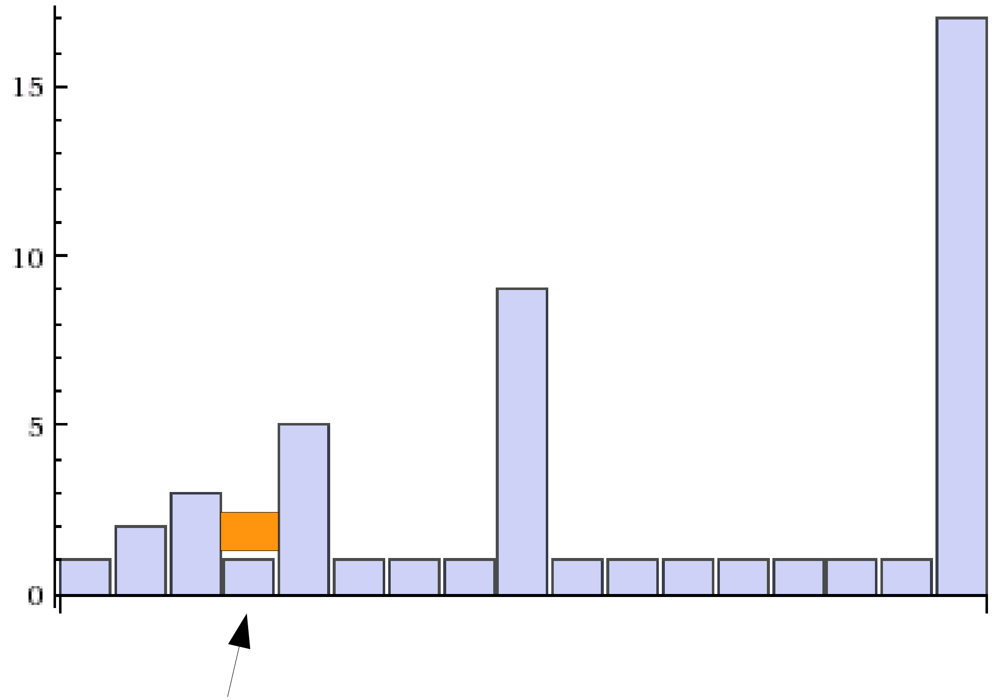


Need 3; have 2+1=3

Amortized Analysis

- How much extra do we charge?



Need 1; have 2; save one!

Amortized Analysis

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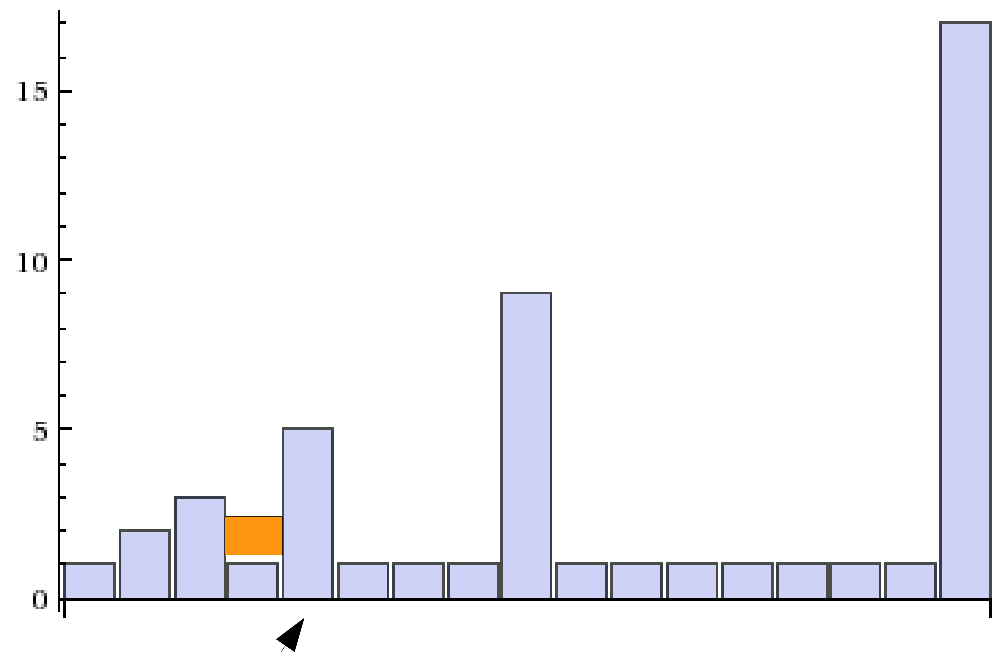
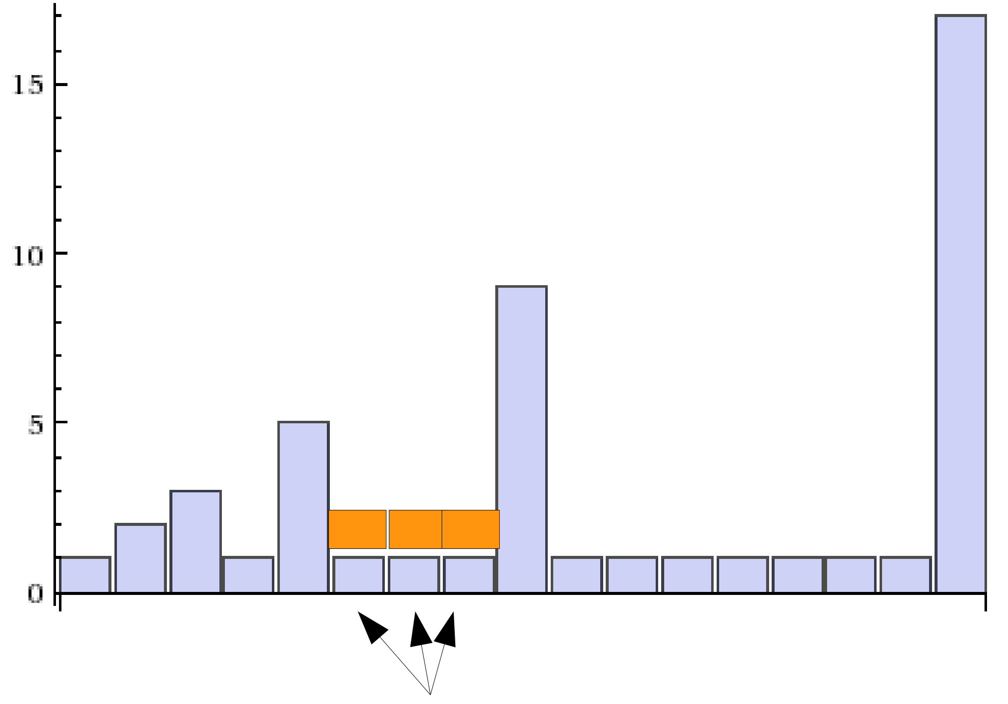


Need 5; have 2+1=3 !!

Amortized Analysis

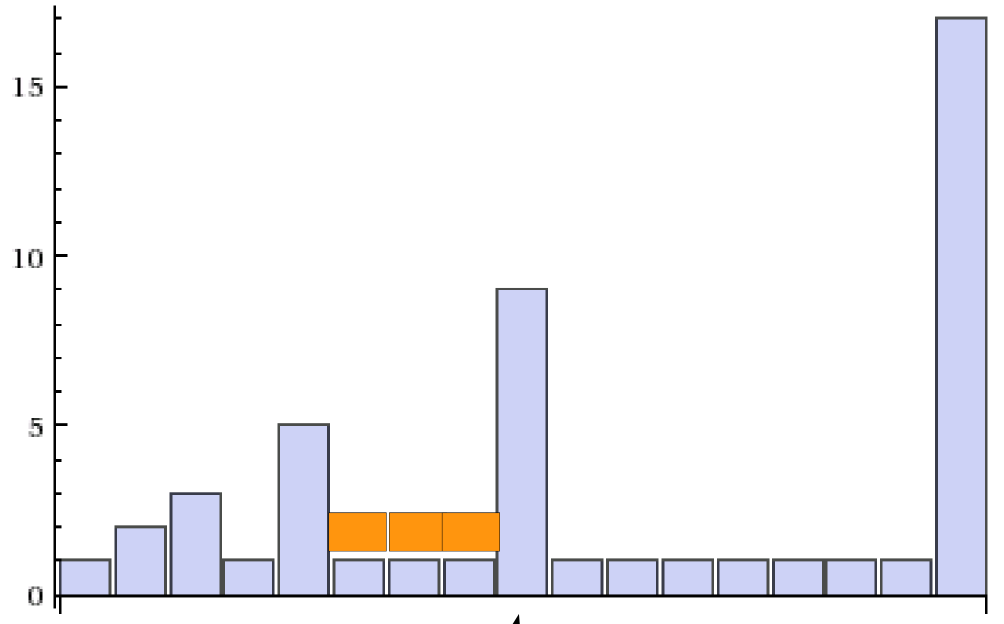
- How much extra do we charge?



Need 1, have 2; save one each!

Amortized Analysis

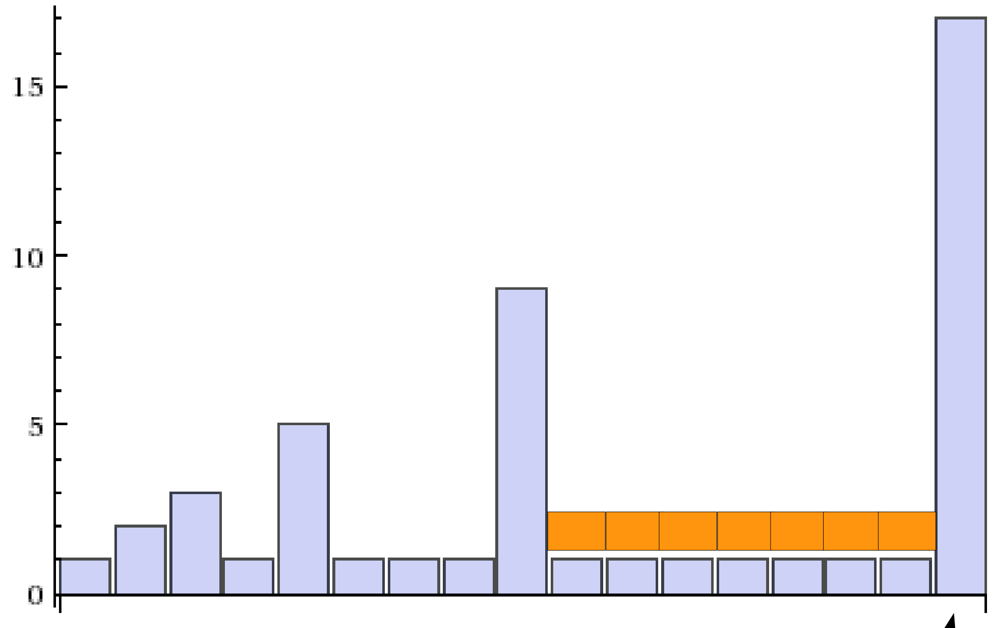
- How much extra do we charge?



Need 9, have 2+3=5 !!

Amortized Analysis

- How much extra do we charge?



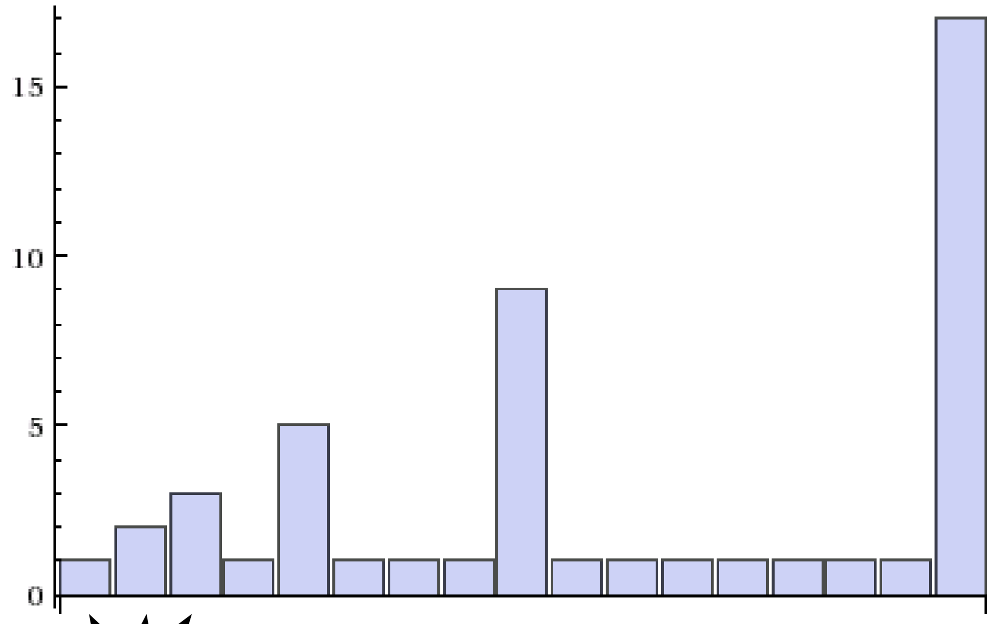
Need 17, have 2+7=9 !!

Amortized Analysis

- How much extra do we charge?
  - Let's try charging 2 extra operations
  - Total of 3 operations per append

Amortized Analysis

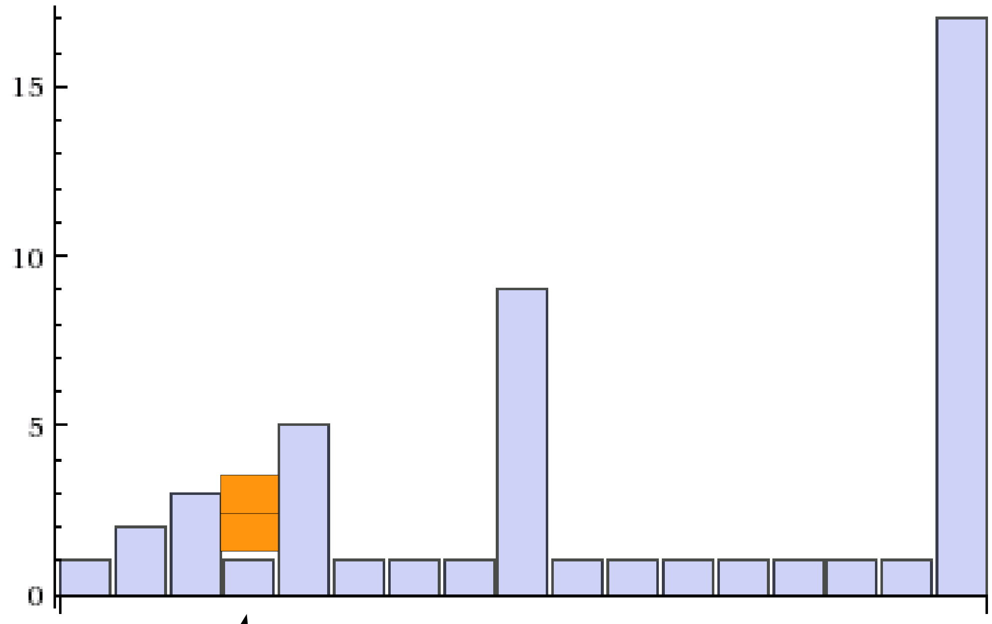
- How much extra do we charge?



Need <=3; have 3

Amortized Analysis

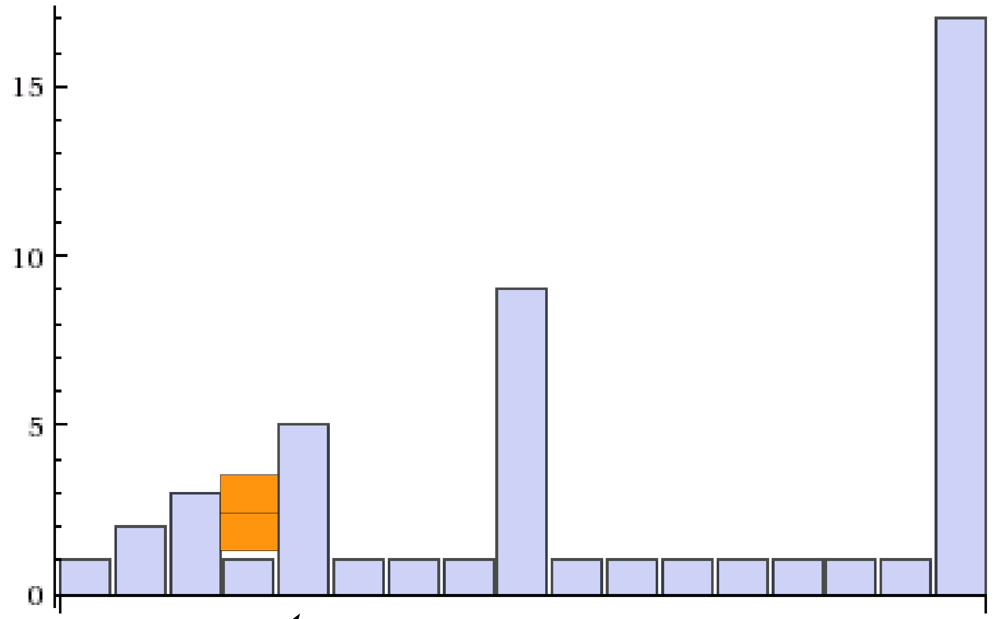
- How much extra do we charge?



Need 1; have 3; save two!

Amortized Analysis

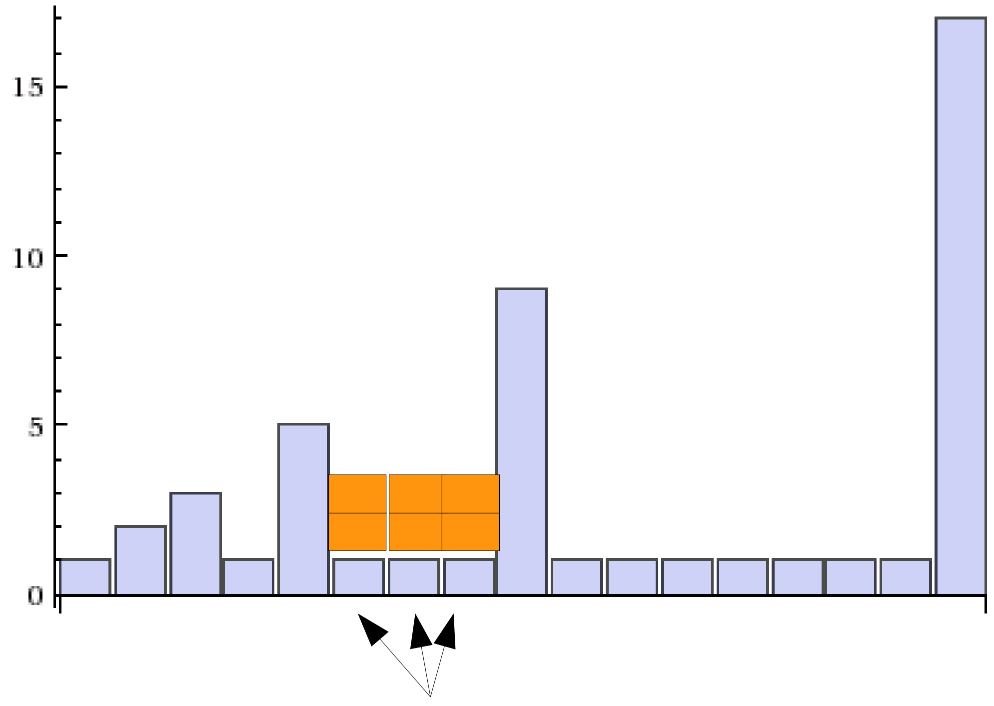
- How much extra do we charge?



Need 5; have 3+2=5

Amortized Analysis

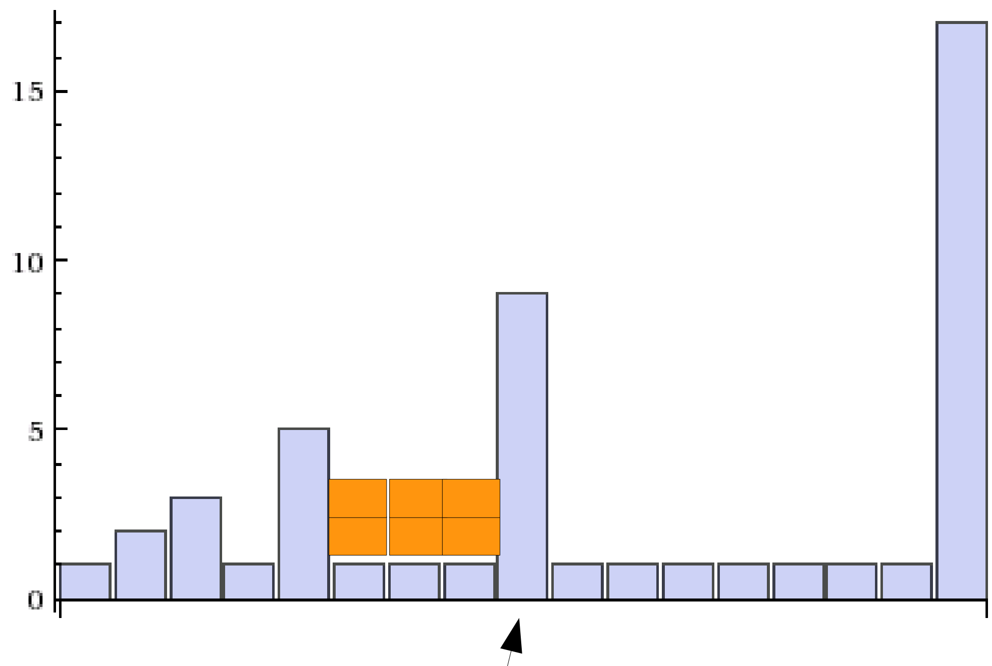
- How much extra do we charge?



Need 1, have 3; save two each!

Amortized Analysis

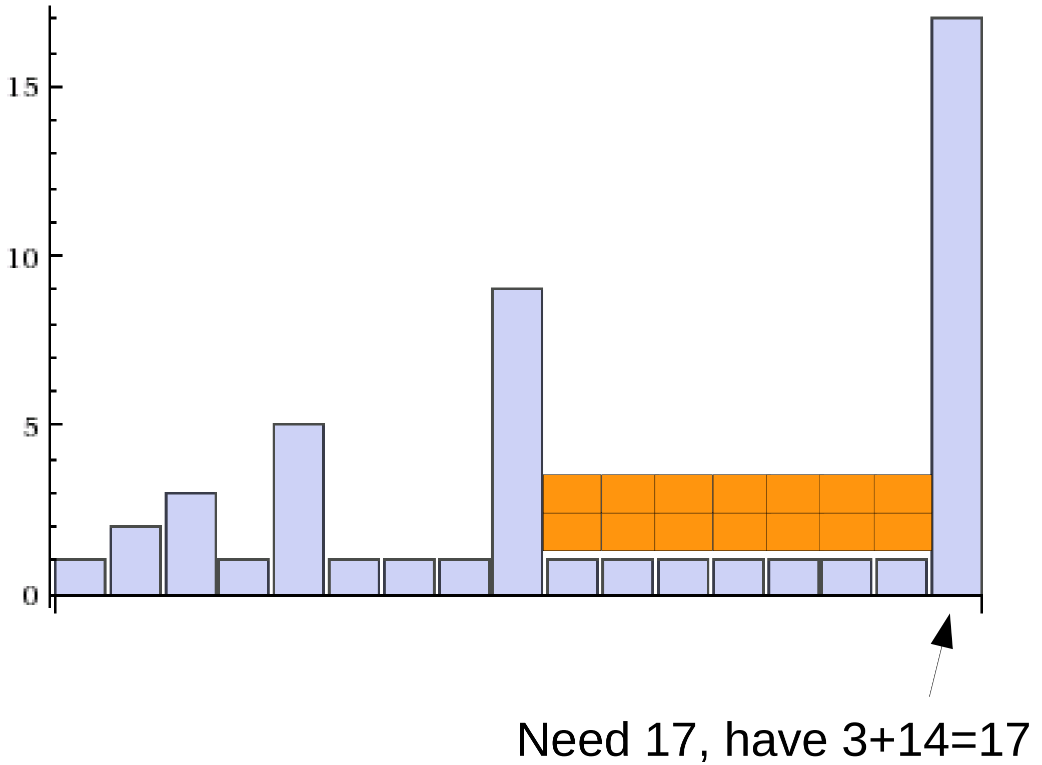
- How much extra do we charge?



Need 9, have 3+6=9

Amortized Analysis

- How much extra do we charge?



Amortized Analysis

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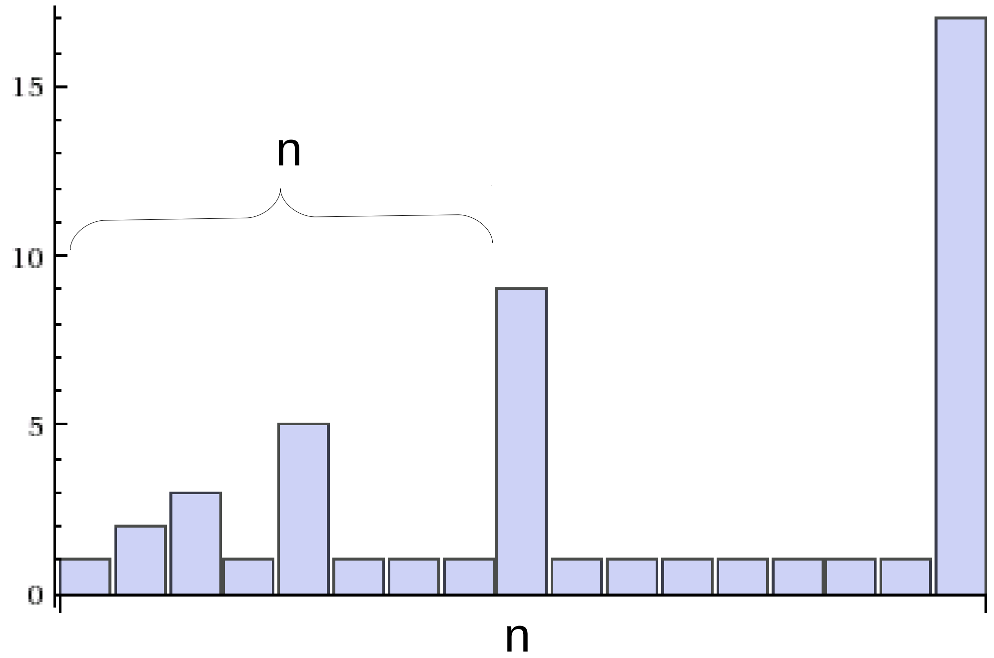

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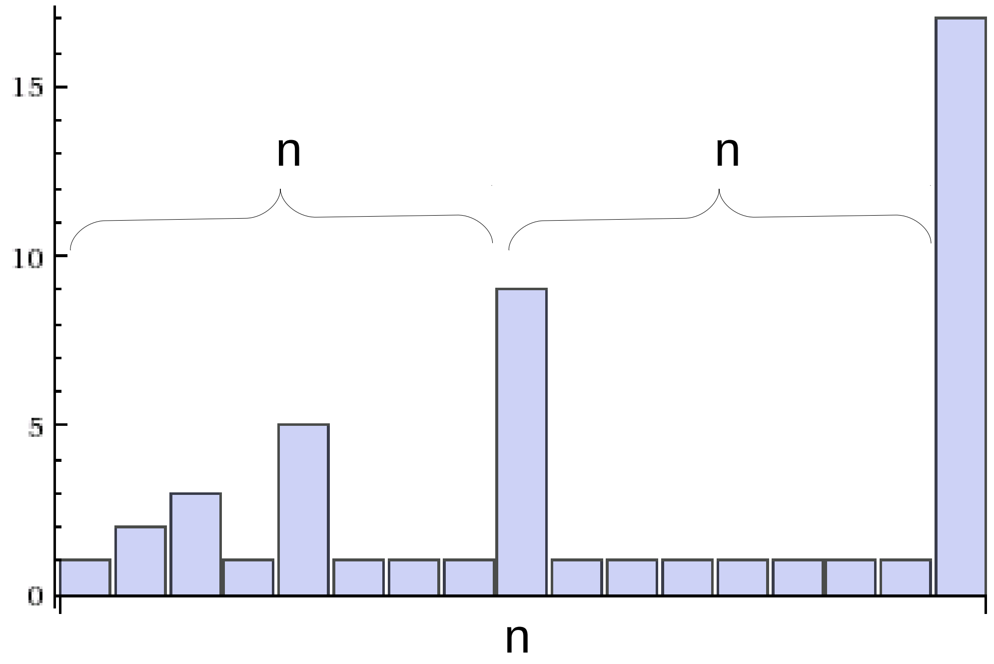

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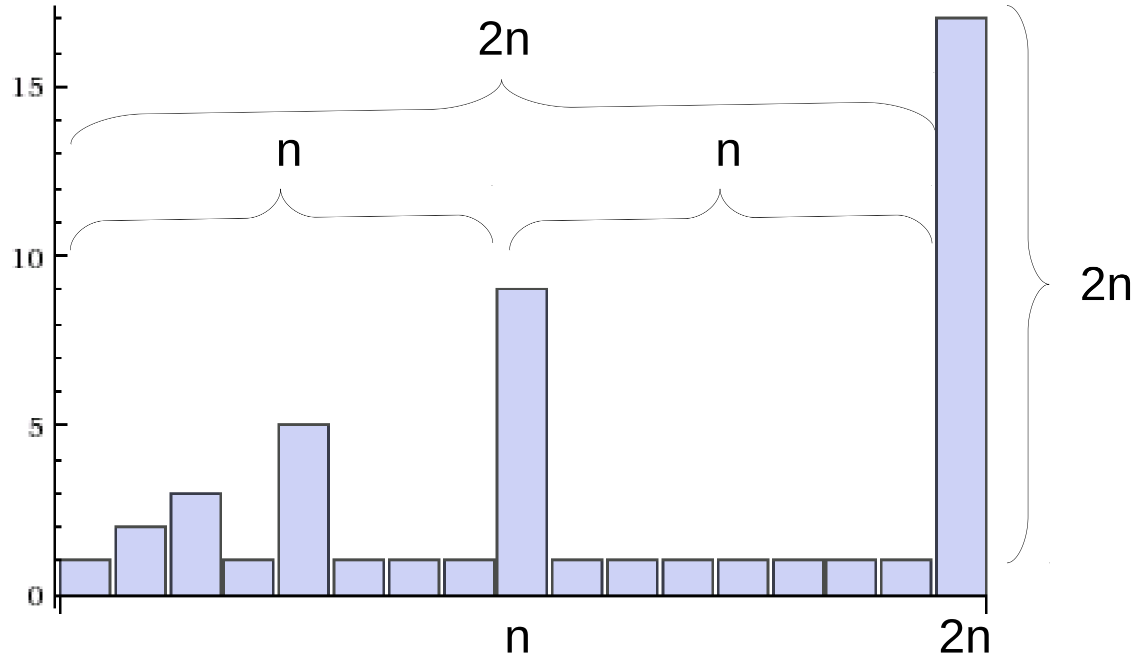

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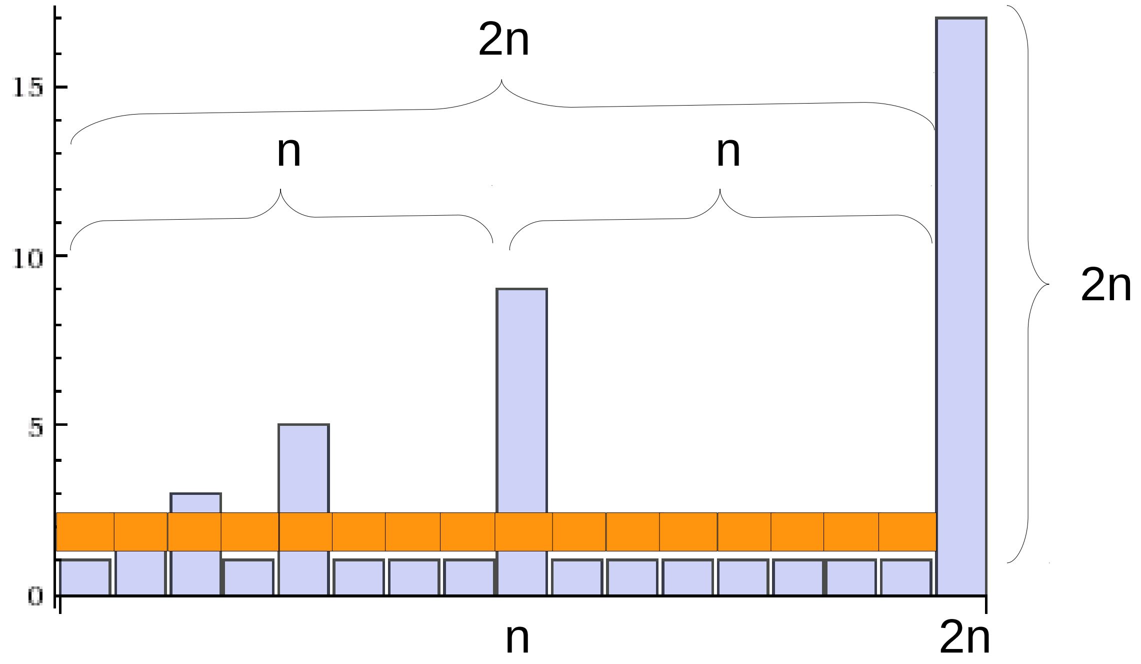

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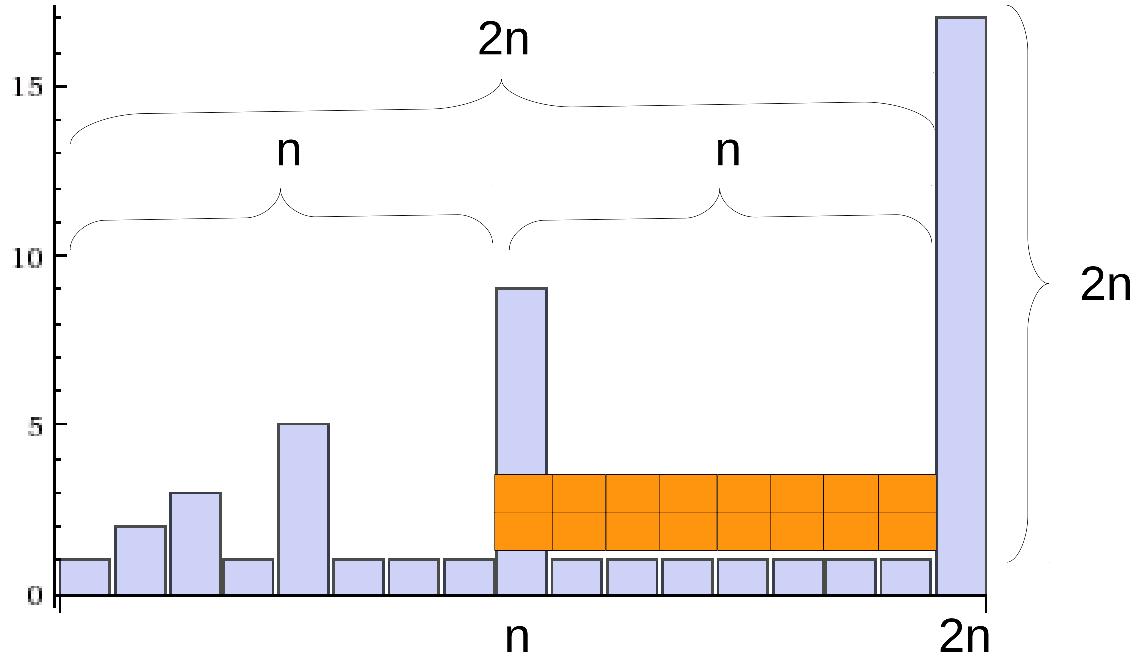

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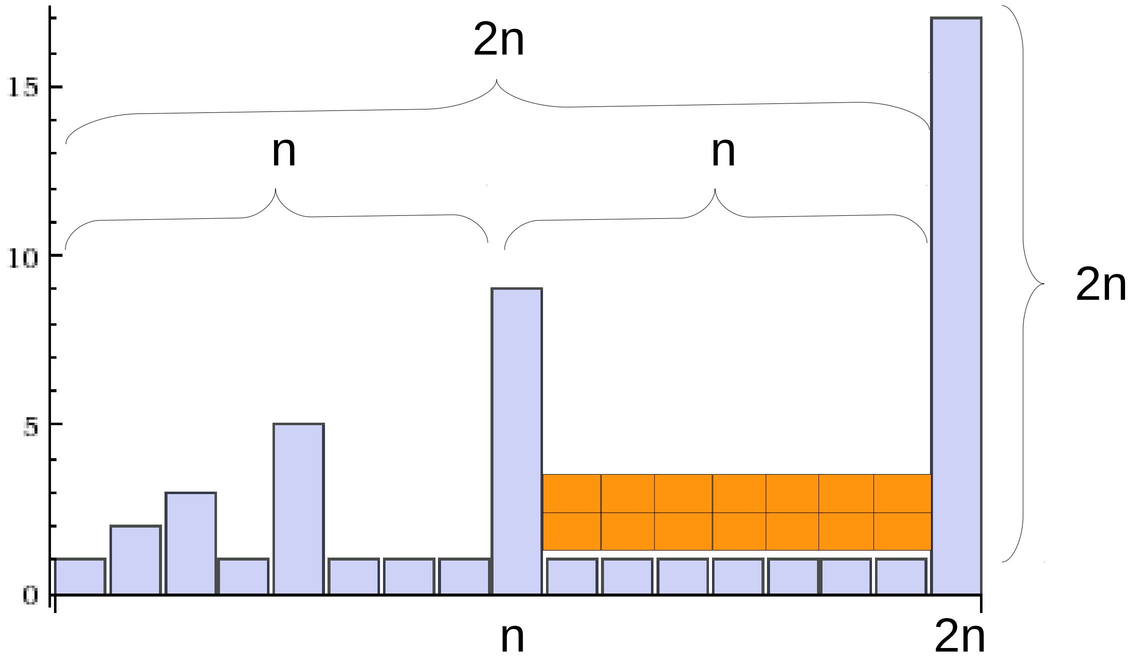


Amortized Analysis

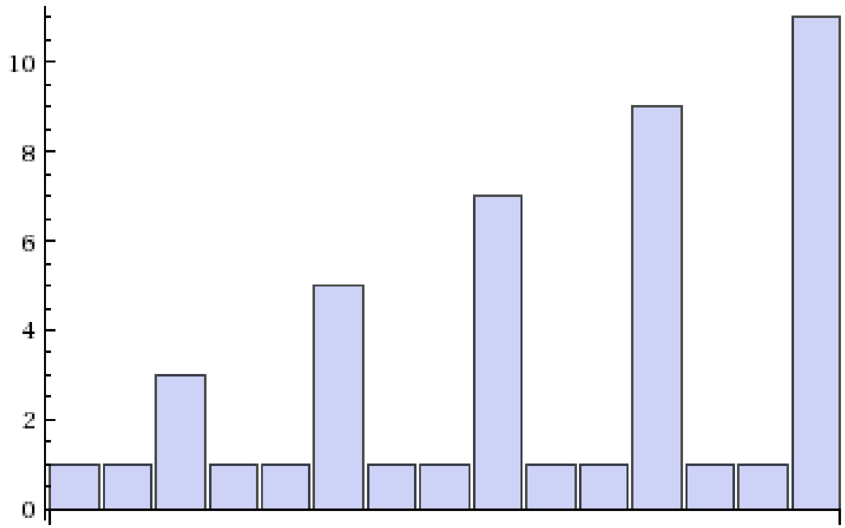
- How much extra do we charge?
  - If we're doubling the size each time…
    - We will need to make 2n copies at the next increase
    - We will have n new appends during that period
  - So we need to “save up” two extra operations per cheap append to pay for the expensive appends
  - Charge 3 total operations for each append

Amortized Analysis

- Total # of operations to add n items: 3n
  - Which is O(n)
- Average operations per append = 3n/n = 3
- More generally: the total # of operations is O(n so the amortized cost per append is O(1)

Amortized Analysis

- Does the same argument apply to a constant increase when the capacity is reached?



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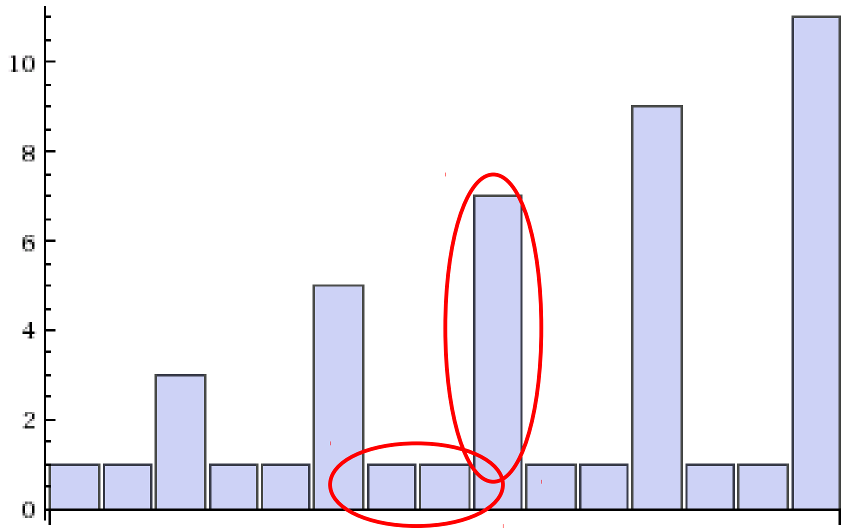

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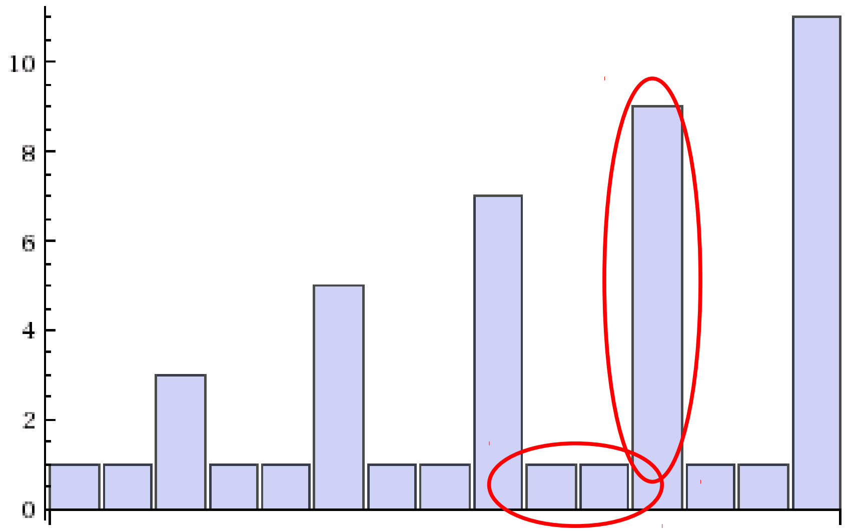

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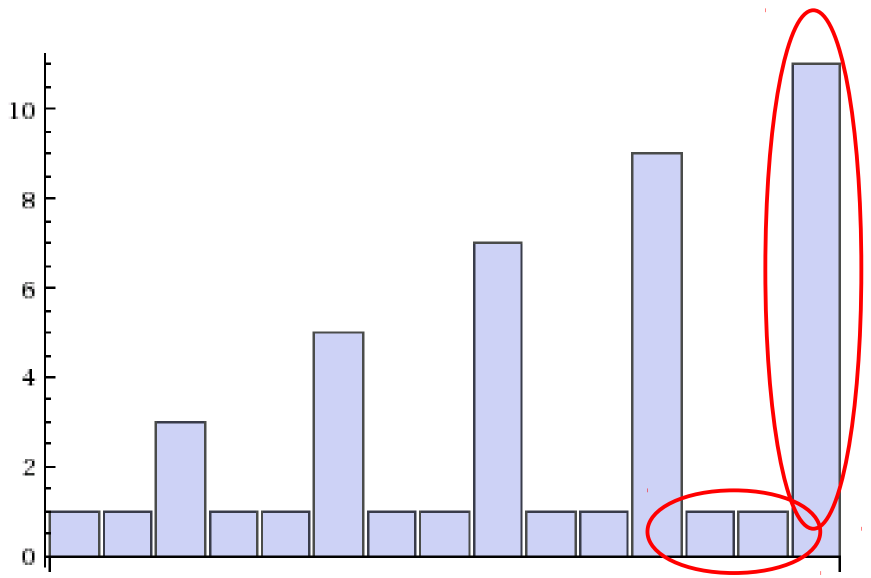
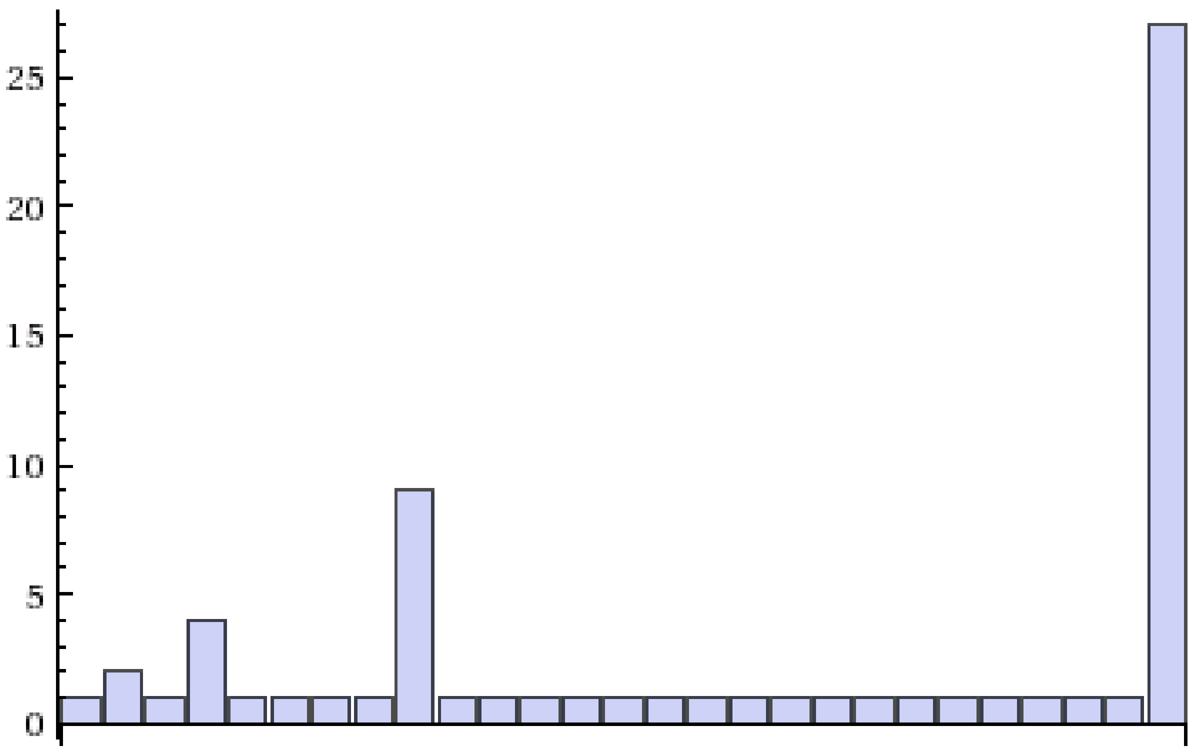


Amortized Analysis

- Does the same argument apply to a constant increase when the capacity is reached?
  - No! The amount of operations "saved" is always constant between increases, but the amount of work done by the capacity increases grows linearly with the size of the array.
  - This actually leads to $\Omega(n^2)$ total operations for n appends, instead of O(n) total operations

Amortized Analysis

- Does the same argument apply to a tripling increase when the capacity is reached?



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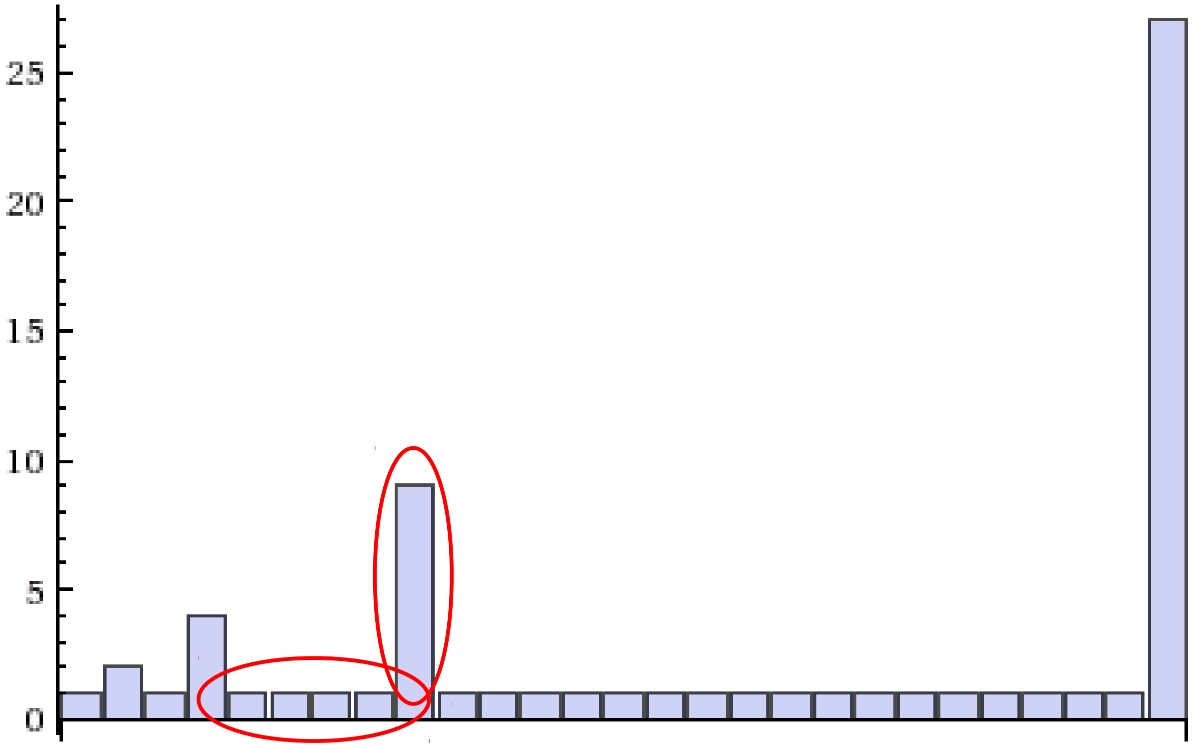

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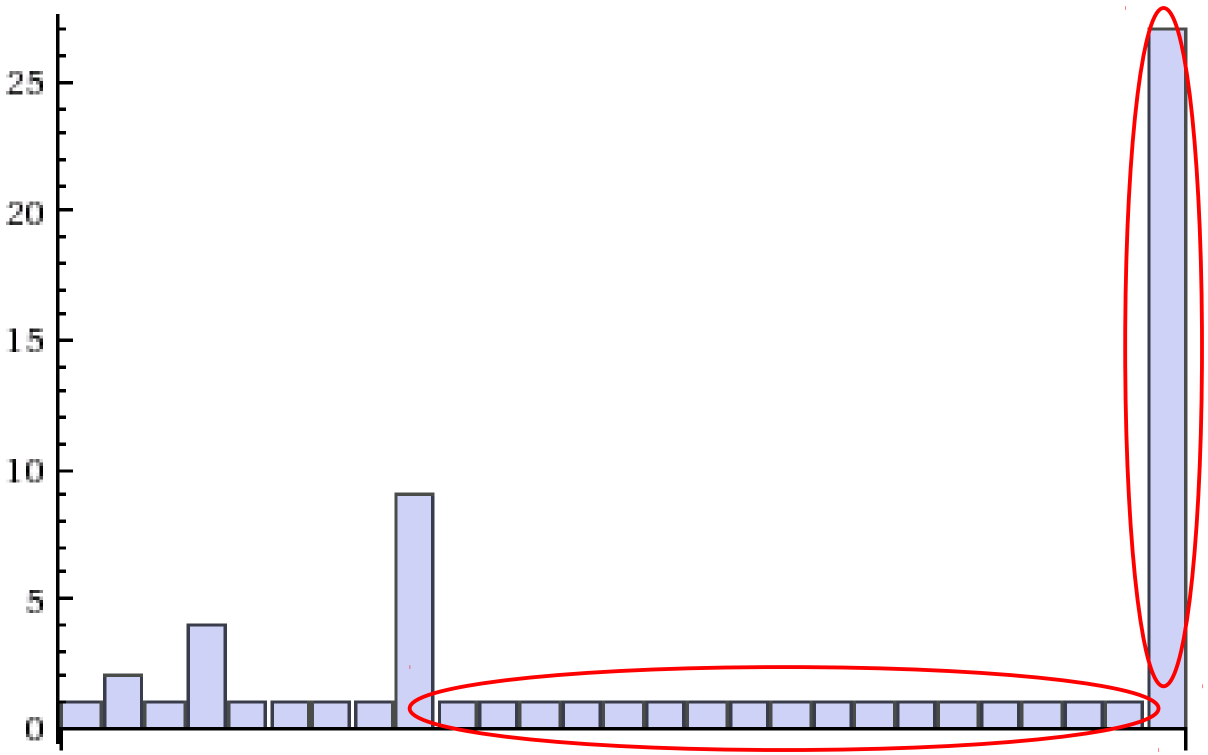

Amortized Analysis

- Does the same argument apply to a tripling increase when the capacity is reached?
  - Yes! Charge three extra operations instead of two, and then we will have saved roughly 3n operations before the next capacity increase.
  - Total operations for *n* appends: $4n \in O(n)$
    - The amortized cost for each append is still O(1)
  - In fact, the argument works for **any** geometric progression

Amortized Analysis

- Python lists don't use strict geometric progression
- But the average cost is still O(1)
- See Section 5.3.3 for evidence

Amortized Analysis

- Overcharge for cheap operations to “save up” credit for expensive operations
- If the total cost for *n* operations can be shown to be O(n), then the average cost for each individual operation is O(1)

Next Time

- Complexity classes for common operations on Python lists and strings