Finite-State Machines with No Output

Section 13.3

Languages

- A vocabulary (or alphabet) *V* is a finite non-empty set of elements called symbols.
- A word (or sentence) over *V* is a string of finite length of elements of *V*.
- The empty string or null string, denoted by λ , is the string containing no symbols.
- The set of all words over V is denoted by V*.
- A language over V is a subset of V*.

Set of Strings

- The *concatenation* of A and B, where A and B are subsets of V^*, denoted by AB, is the set of all strings of the form xy, where x is a string in A and y is a string in B.

- Let $A = \{0, 11\}$ and $B = \{1, 10, 110\}$. Then

  $$AB = \{01, 010, 0110, 111, 110, 11110\}$$

  and

  $$BA = \{10, 111, 100, 1011, 1100, 11011\}$$

Set of Strings

- If A is a subset of V^*, the *Kleene closure* of A, denoted by A^*, is the set consisting of arbitrarily long strings of elements of A. That is,

$$A^* = \bigcup_{k=0}^{\infty} A^k$$

- The Kleene closures of the sets $A = \{0\}$, $B = \{0,1\}$ and $C = \{11\}$ are

$$A^* = \{0^n \mid n = 0, 1, 2, \ldots.\}$$

$$B^* = V^*$$

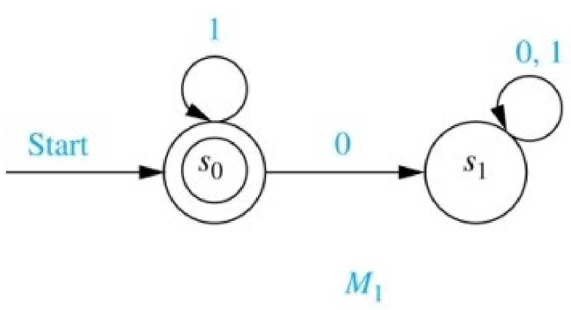
$$C^* = \{1^{2n} \mid n = 0, 1, 2, \ldots.\}$$

Language Recognition by FSAs

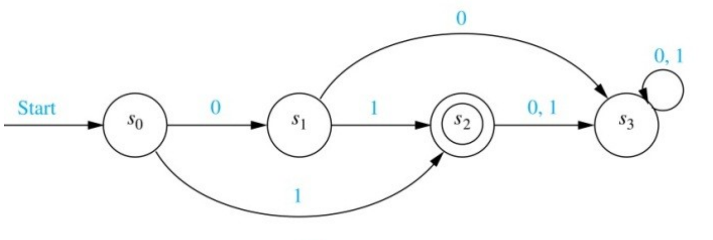
- A string x is said to be *recognized* (or *accepted*) by the machine $M = (S, I, f, s_0, F)$ if it takes the initial state s_0 to a final state, that is, $f(s_0, x)$. The *language recognized* (or *accepted*) by M, denoted by $L(M)$, is the set of all strings that are recognized by M. Two finite-state automata are called *equivalent* if they recognize the same language.

- The only final state of M_1 is s_0. The strings that take s_0 to itself consist of zero or more consecutive 1s. Hence,

  $L(M_1) = \{1^n \mid n = 0, 1, 2, \ldots\}$.

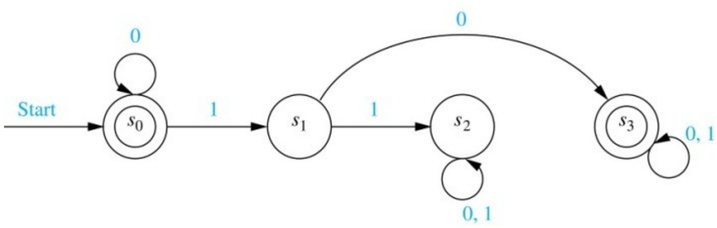


Language Recognition by FSAs



M_2

- The only final state of M_2 is s_2. The strings that take s_0 to s_2 are 1 and 01 . Hence , $L(M_2) = \{1, 01\}$.



M_3

- The final state of M_3 are s_0 and s_3. The strings that take s_0 to itself are λ, 0, 00, 000,… . The strings that take s_0 to s_3 are a string of zero or more consecutive 0s, followed by 10, followed by any string. Hence, $L(M_3) = \{0^n, 0^n 10x \mid n = 0, 1, 2, \ldots,$ and x is any string$\}$

Language Recognition by FSAs (*cont.*)

Example: Construct a FSA that recognizes the set of bit strings that begin with two 0s.

Language Recognition by FSAs (*cont.*)

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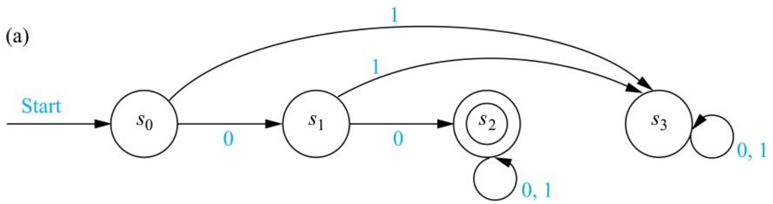


NDFSA

- A nondeterministic finite-state automaton $M = (S, I, f, s_0, F)$ consists of a finite set S of *states*, a finite *input alphabet* I, a *transition function* f that assigns a set of states to every pair of state and input (so that $f: S \times I \rightarrow P(S)$), an *initial* or *start state* s_0, and a subset F of S consisting of *final* (or *accepting*) *states*.

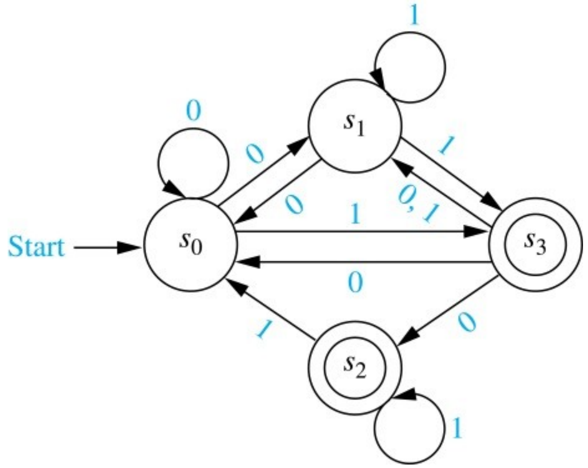
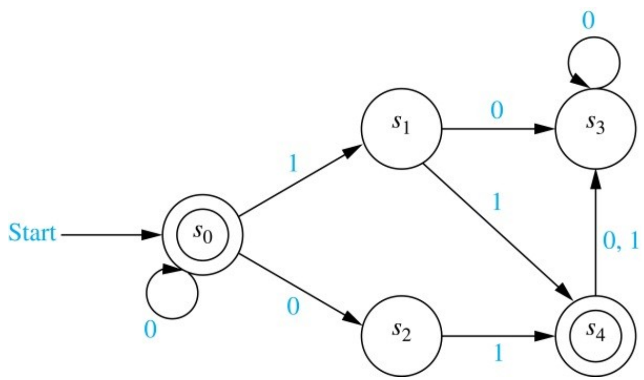


TABLE 2

State	f Input 0	f Input 1
s_0	s_0, s_1	s_3
s_1	s_0	s_1, s_3
s_2		s_0, s_2
s_3	s_0, s_1, s_2	s_1

Finding an Equivalent DFSA

Example: Find a DFSA that recognizes the same language as the NFSA:



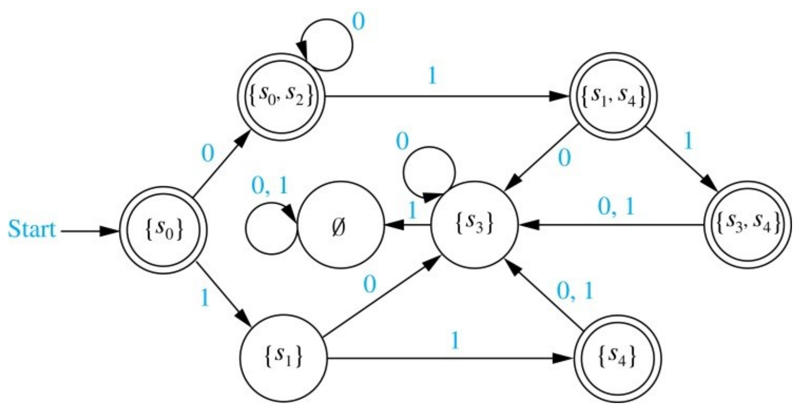


TABLE 3

	f	
	Input	
State	**0**	**1**
s_0	s_0, s_2	s_1
s_1	s_3	s_4
s_2		s_4
s_3	s_3	
s_4	s_3	s_3